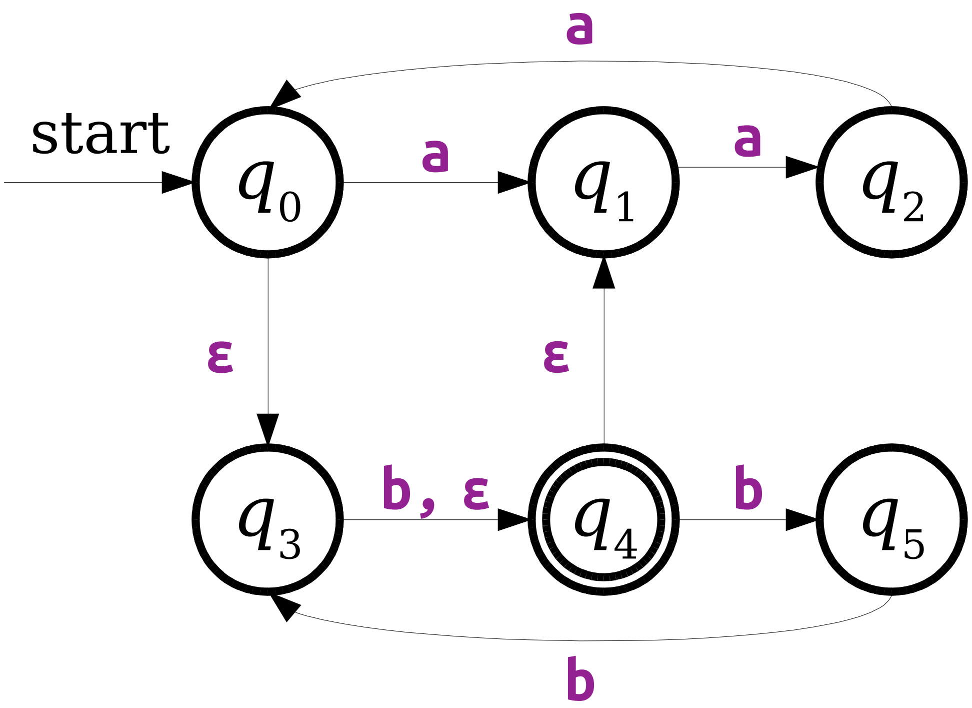
Finite Automata

Part Three

Recap from Last TIme

NFAs

- An ***NFA*** is a
  - ***N***ondeterministic
  - ***F***inite
  - ***A***utomaton
- NFAs have no restrictions on how many transitions are allowed per state.
- They can also use ε-transitions.
- An NFA accepts a string *w* if there is some sequence of choices that leads to an accepting state.



Massive Parallelism

- An NFA can be thought of as a DFA that can be in many states at once.
- At each point in time, when the NFA needs to follow a transition, it tries all the options at the same time.
- The NFA accepts if *any* of the states that are active at the end are accepting states. It rejects otherwise.

New Stuff!

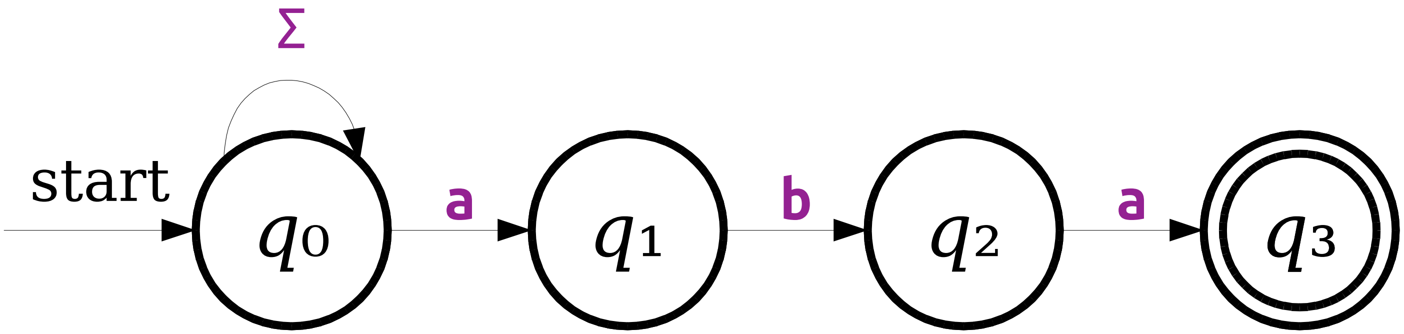
Just how powerful *are* NFAs?

NFAs and DFAs

- Any language that can be accepted by a DFA can be accepted by an NFA.
- Why?
    - Every DFA essentially already *is* an NFA!
- ***Question****:* Can any language accepted by an NFA also be accepted by a DFA?
- Surprisingly, the answer is ***yes***!

Thought Experiment:

How would you simulate an NFA in software?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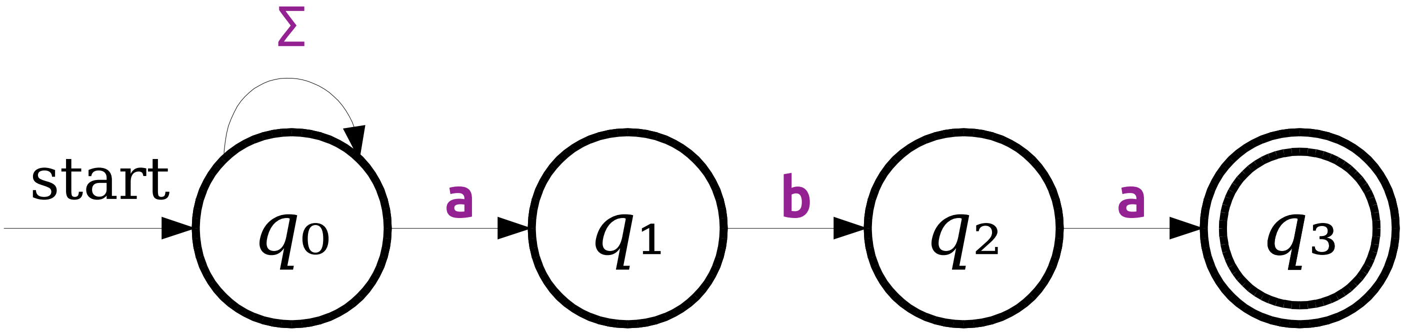

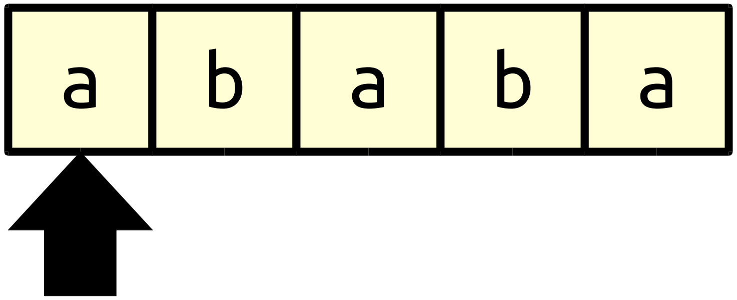

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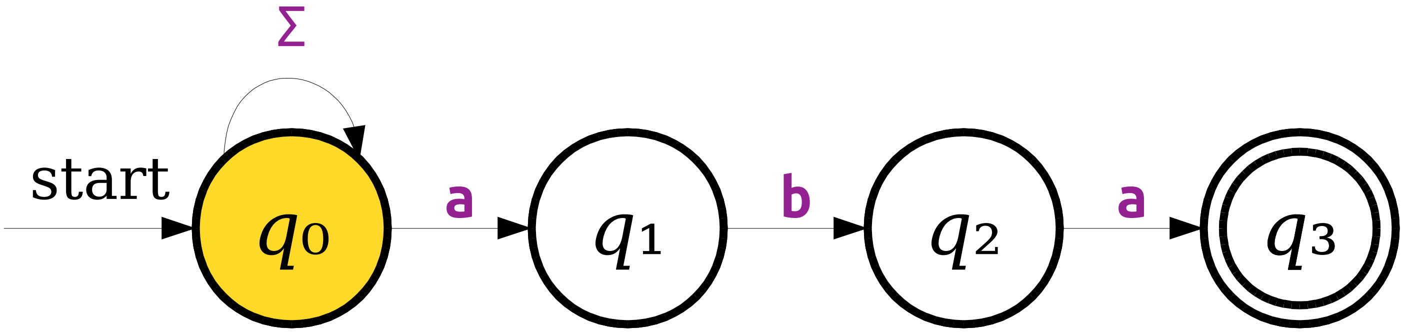
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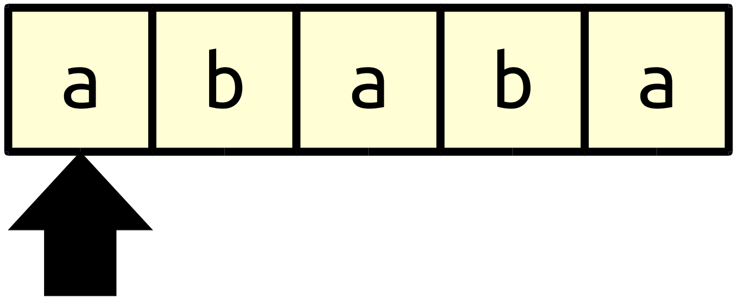
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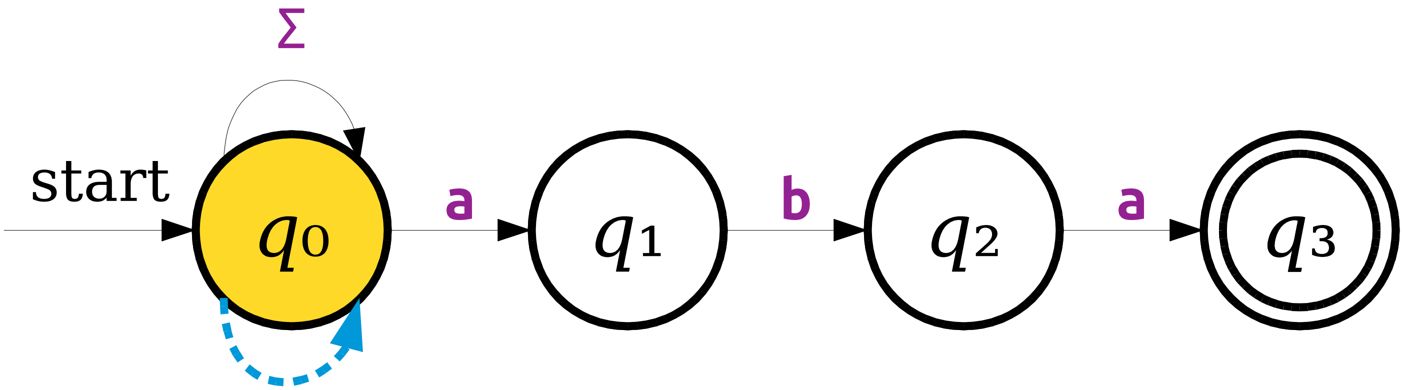

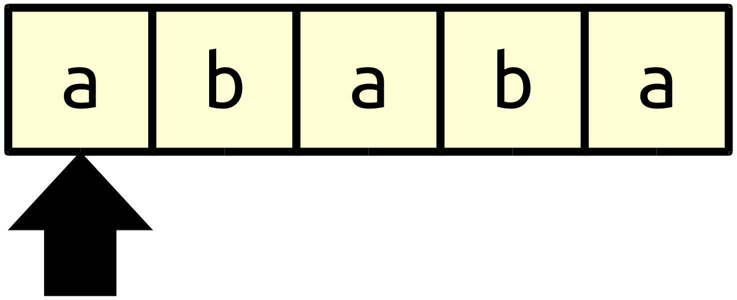

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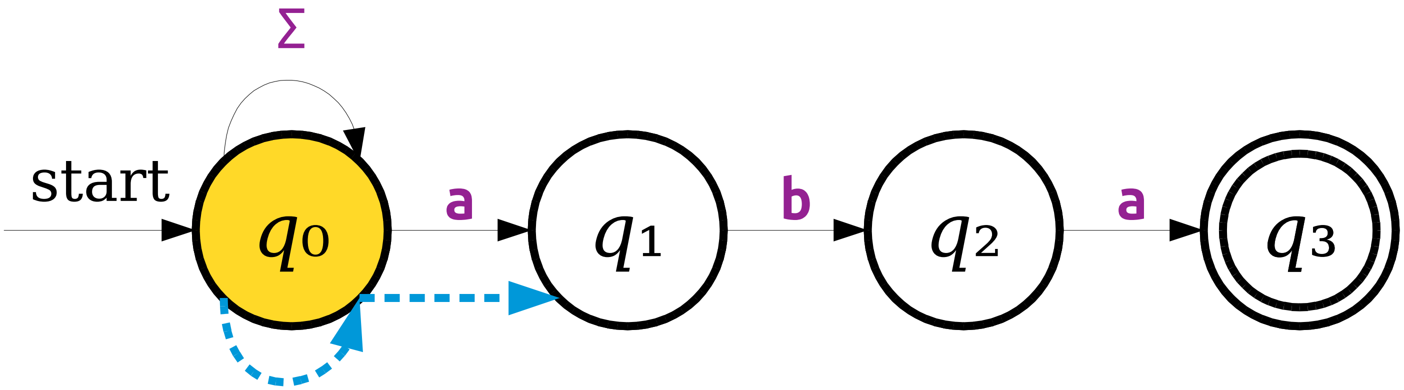
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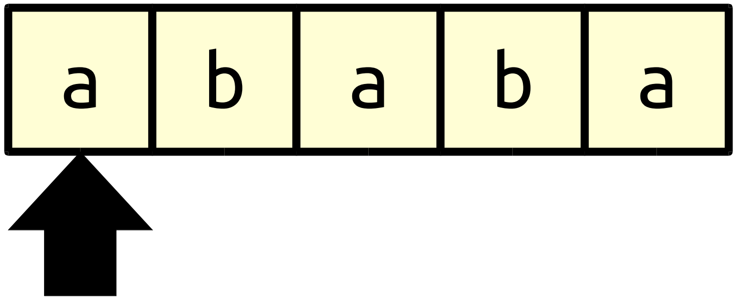
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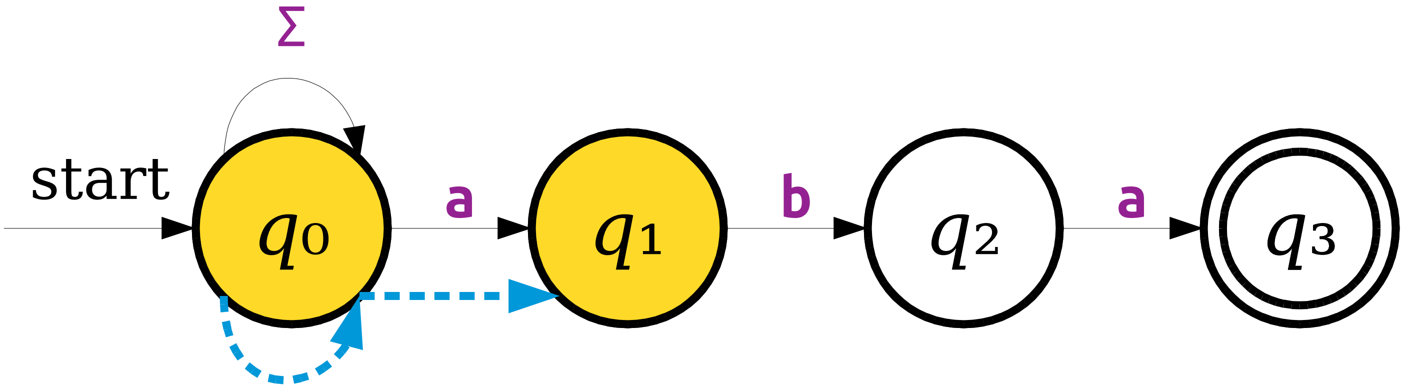
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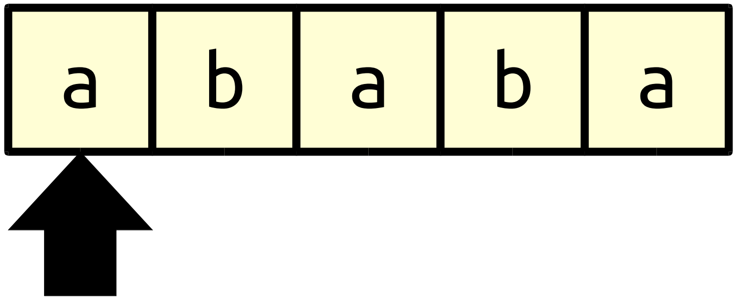
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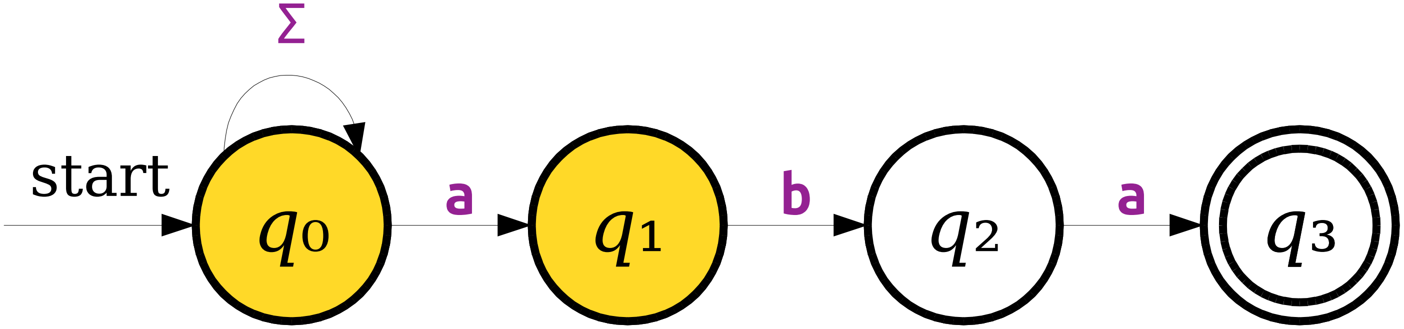

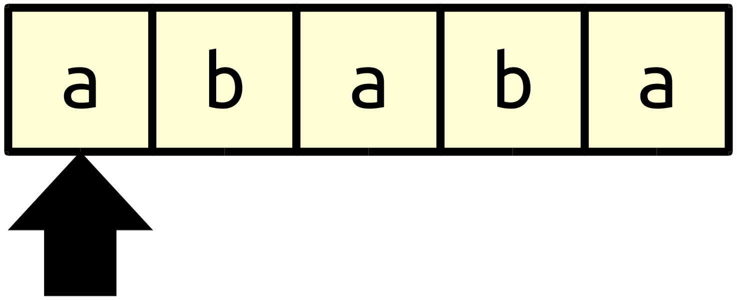

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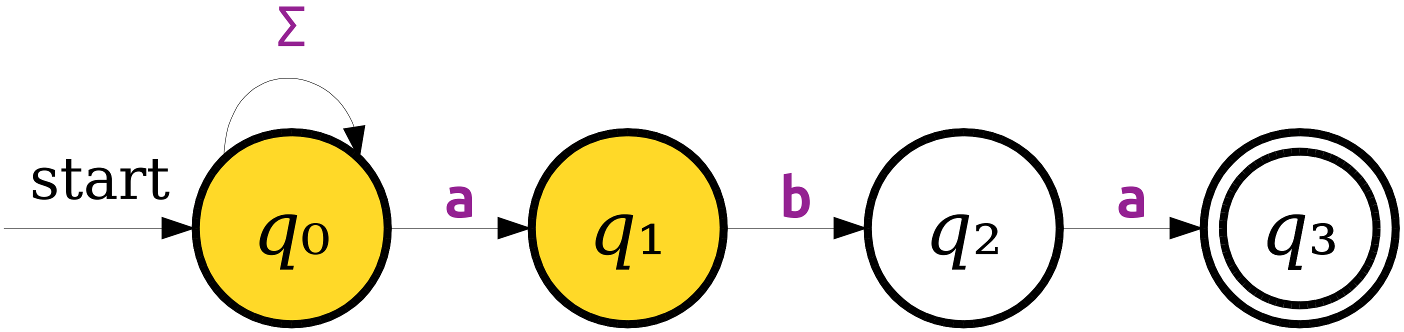
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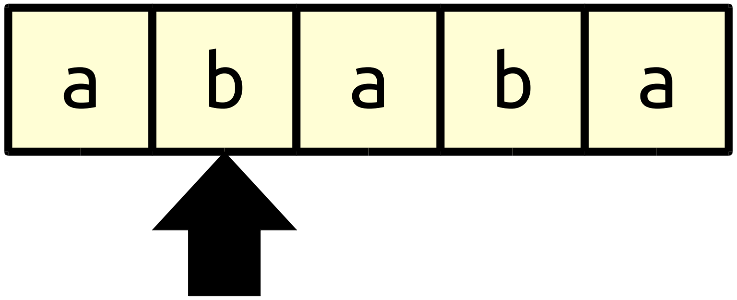
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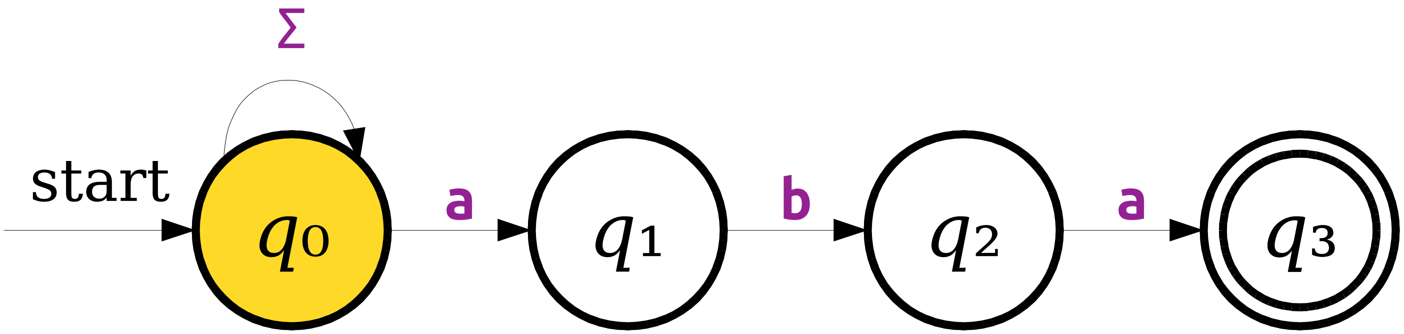

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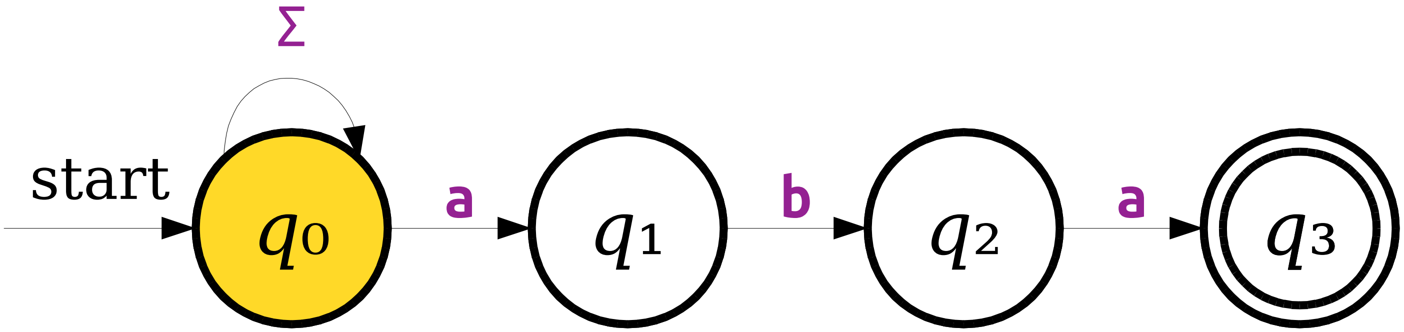
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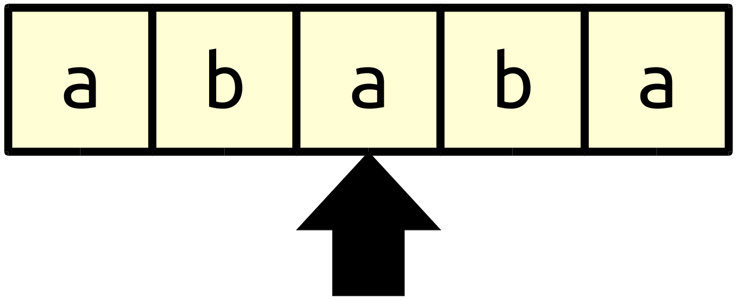
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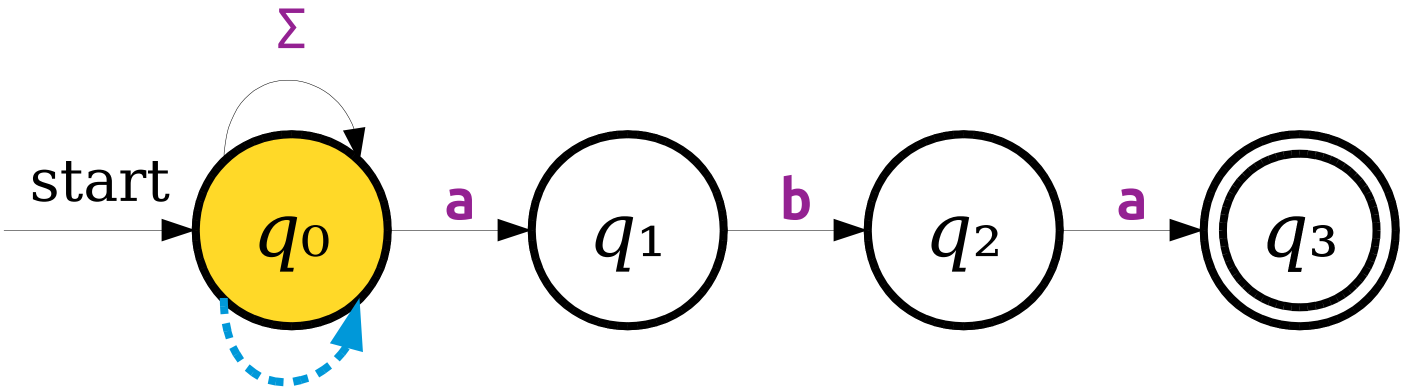
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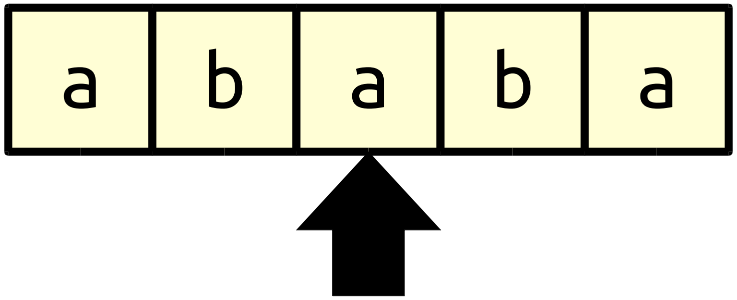
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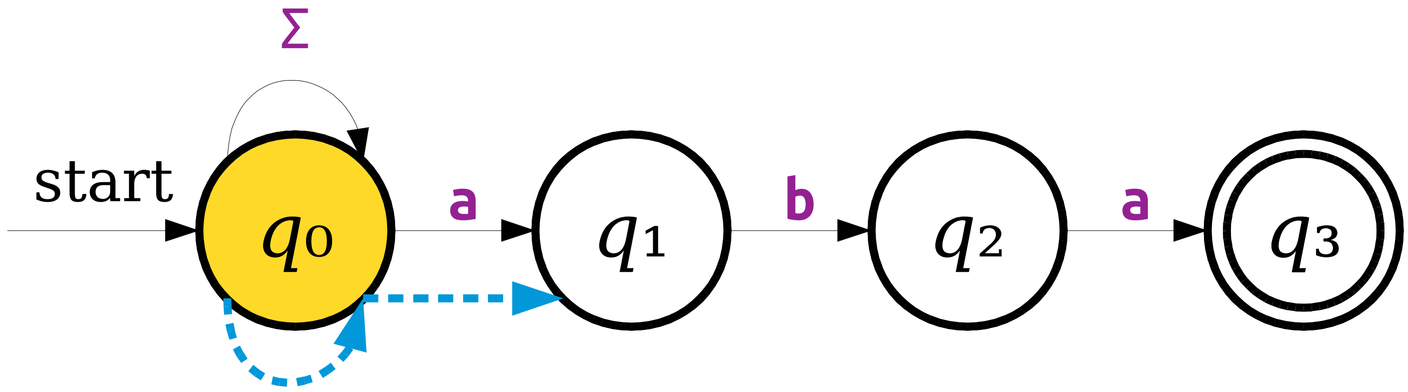
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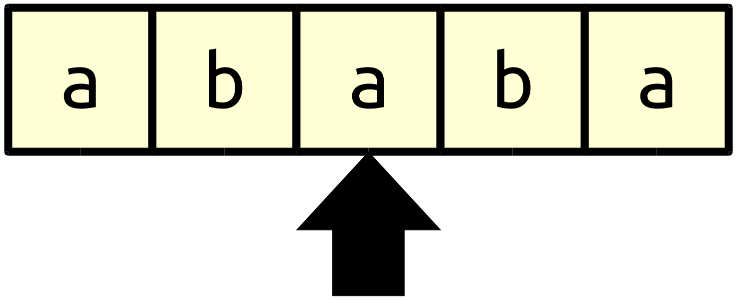
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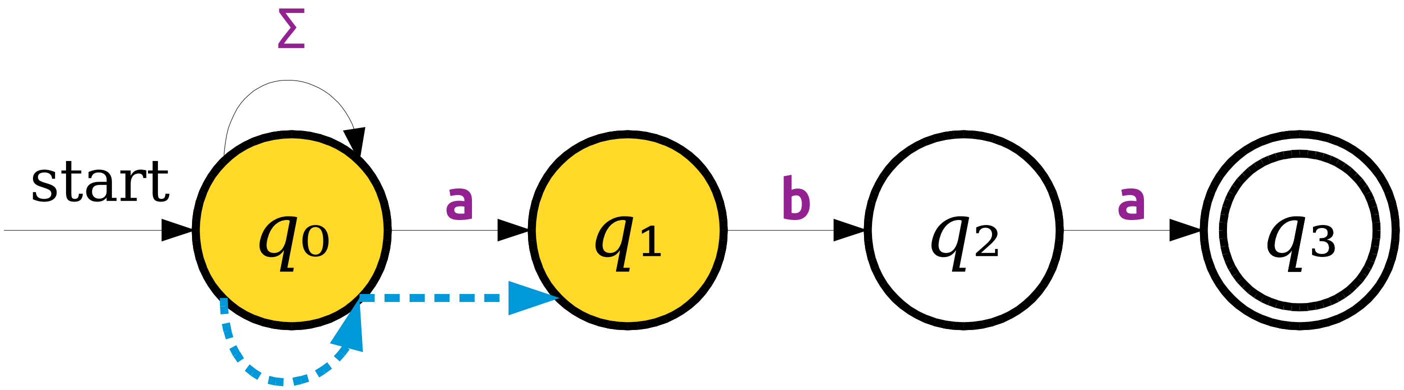
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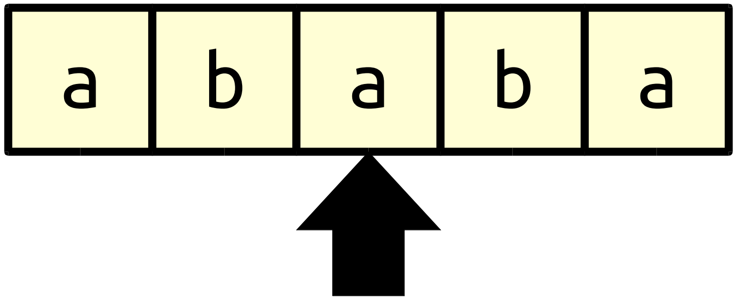
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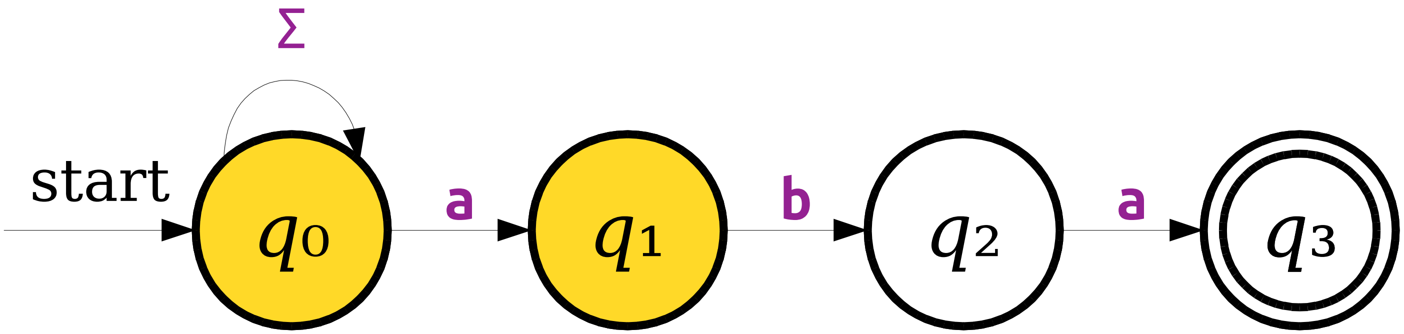
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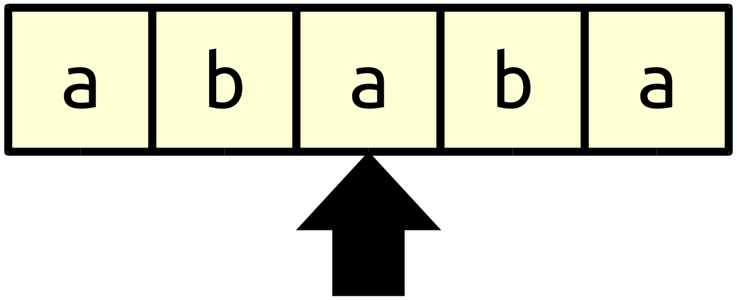
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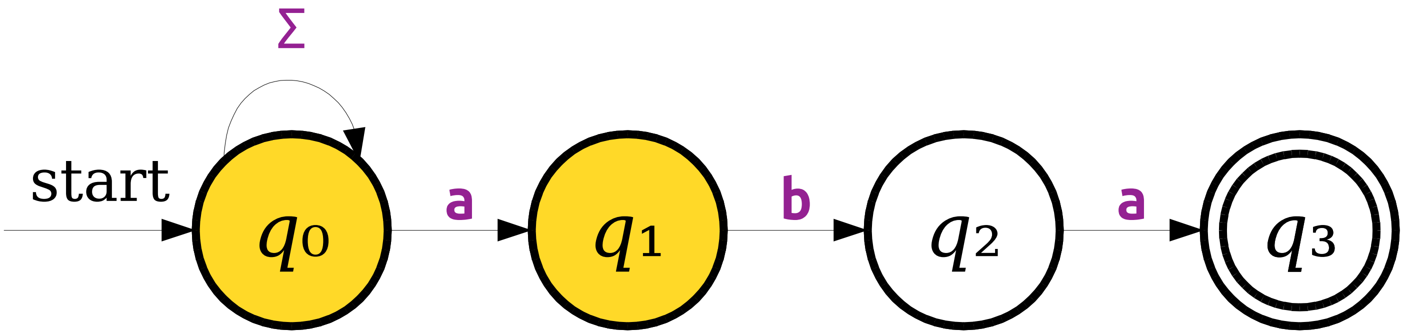
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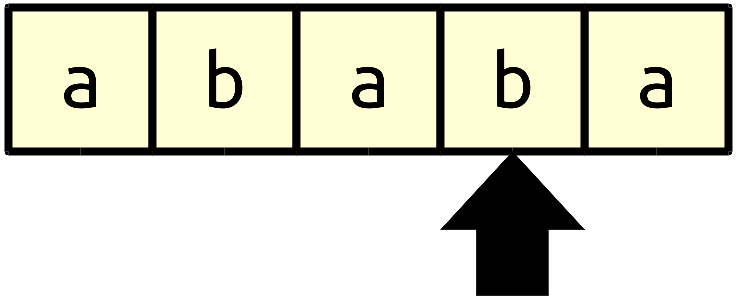
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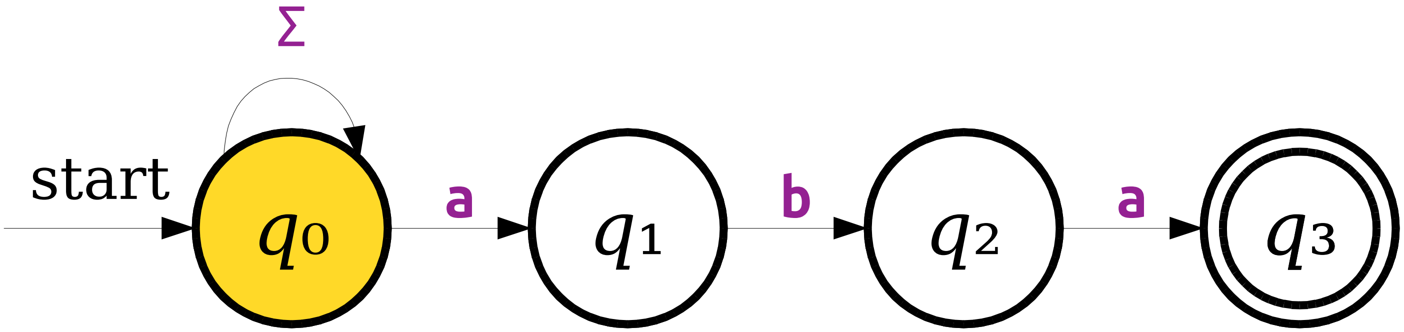

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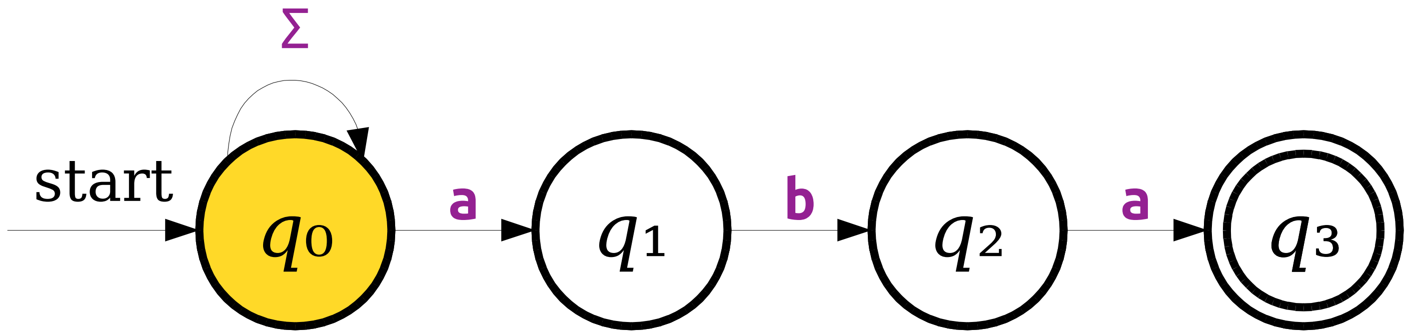
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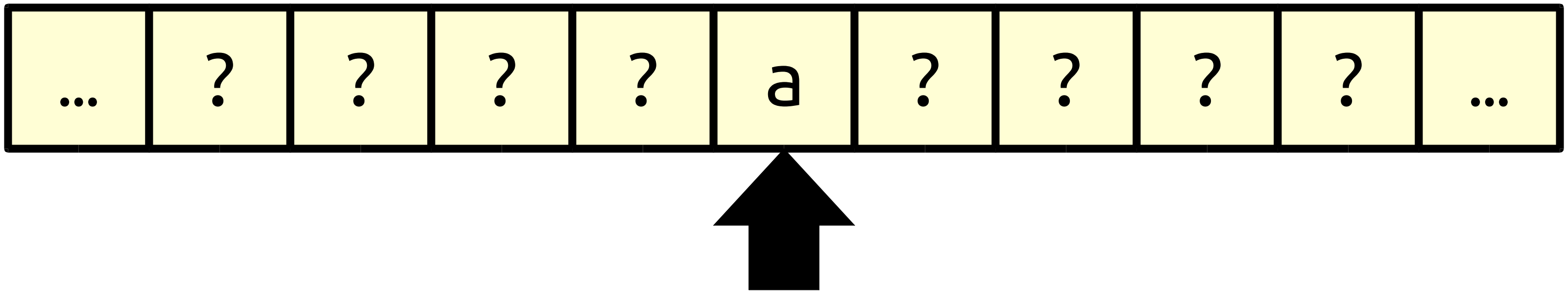
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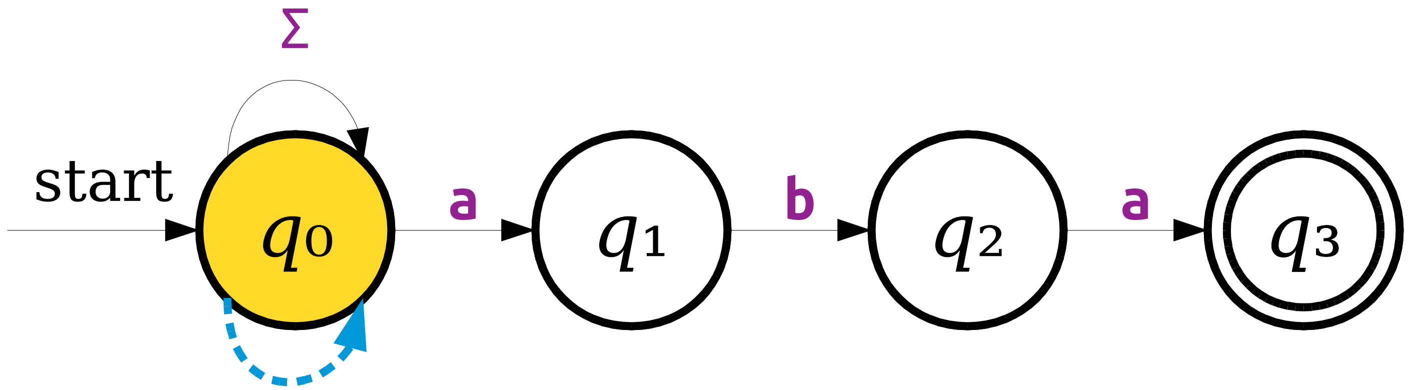

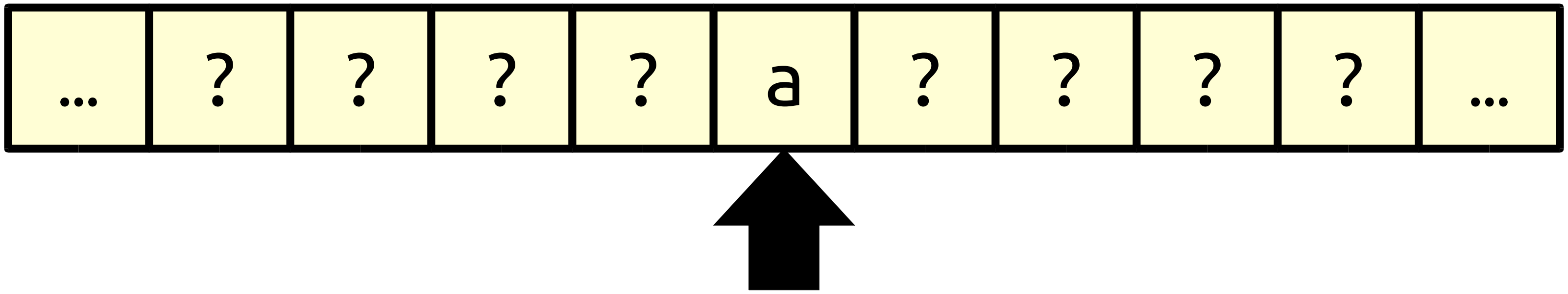

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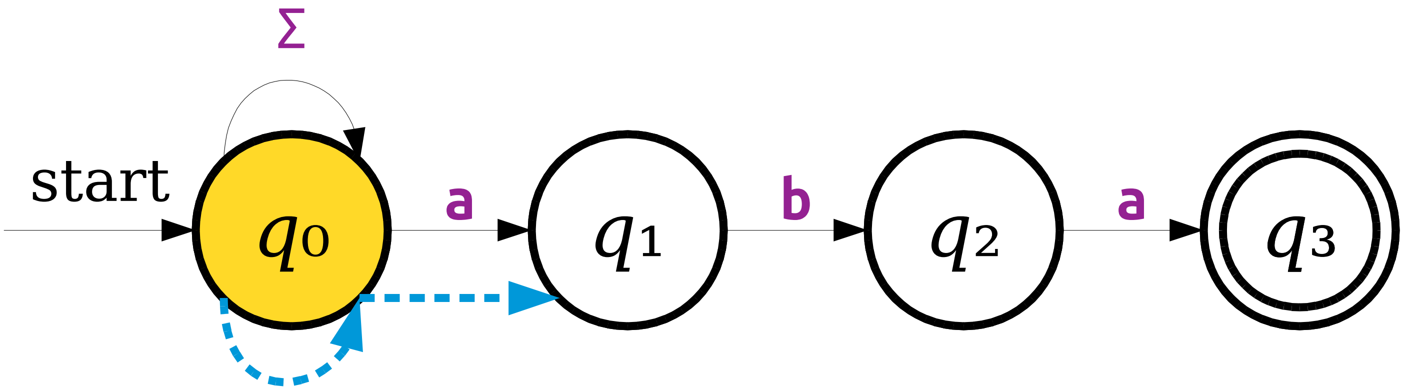

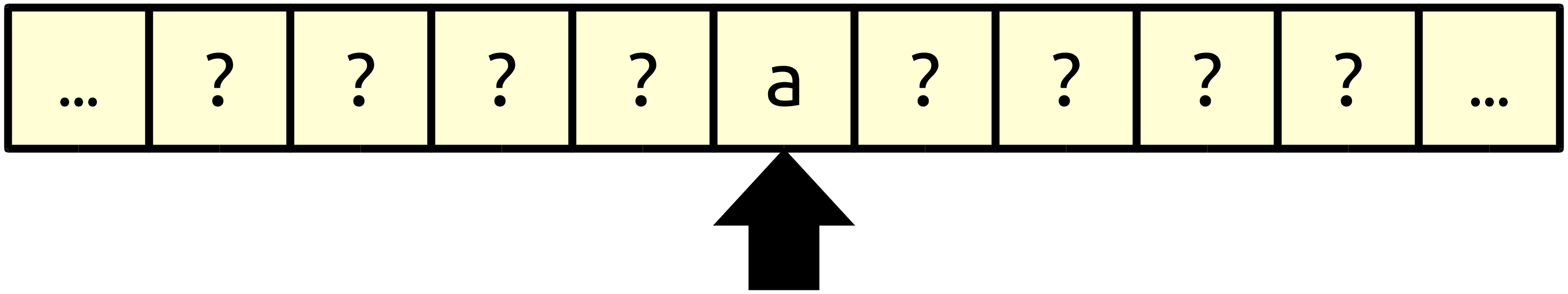

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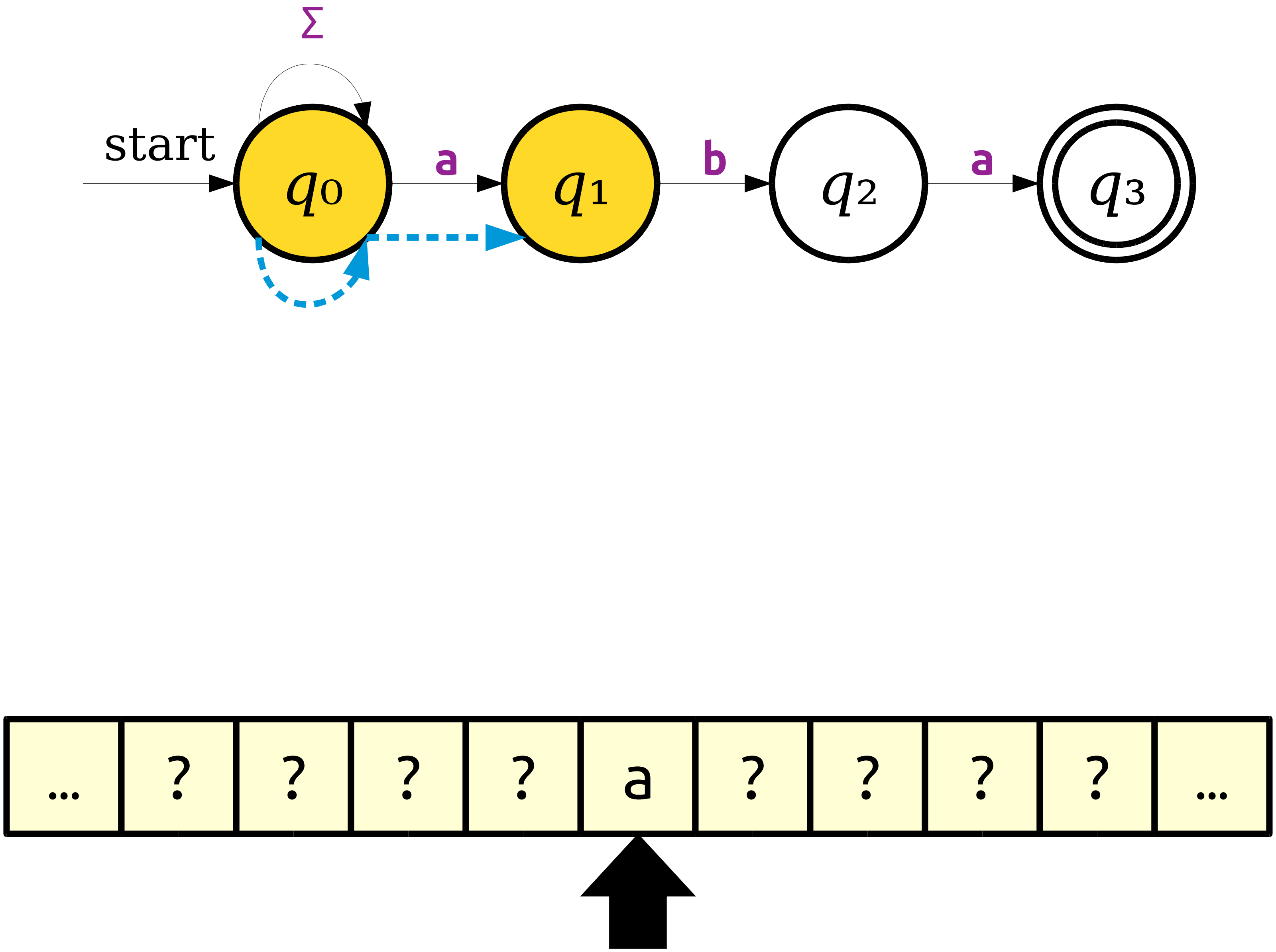
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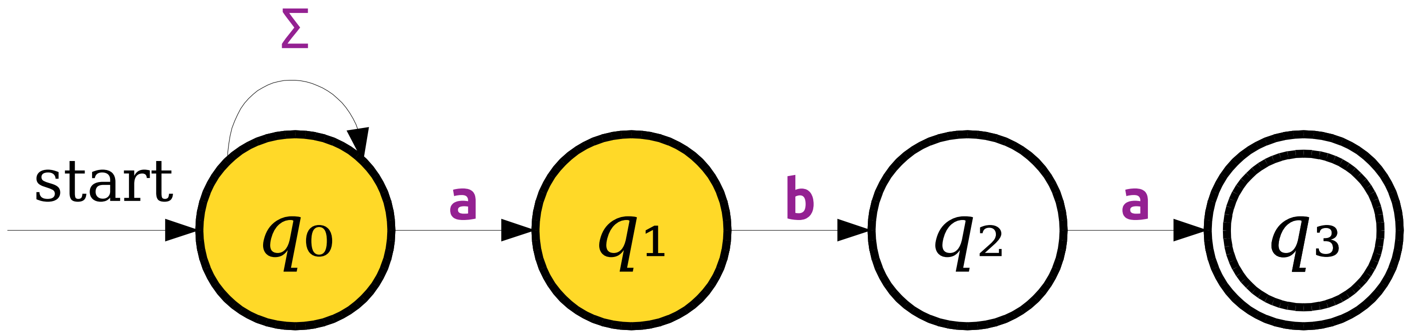

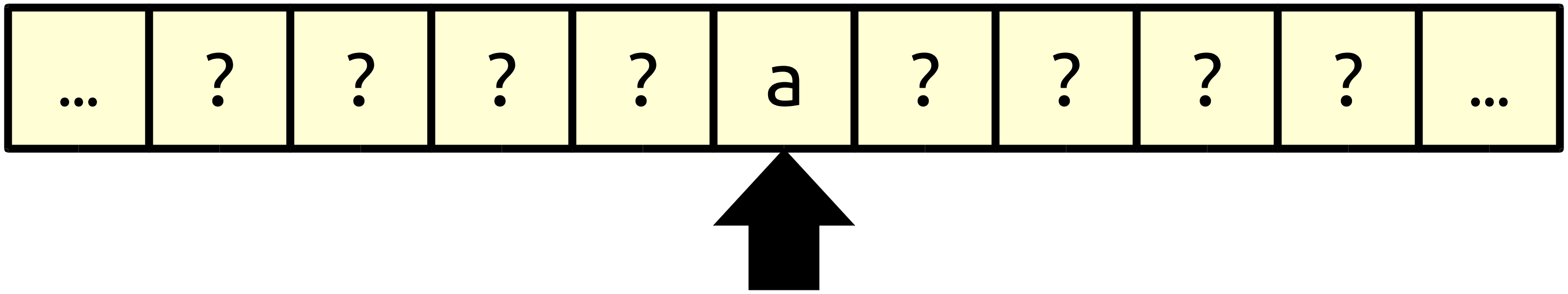

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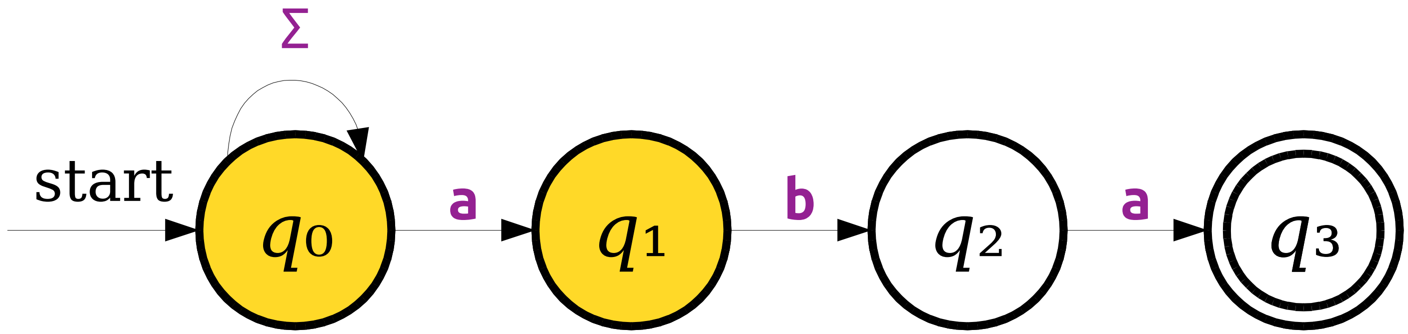

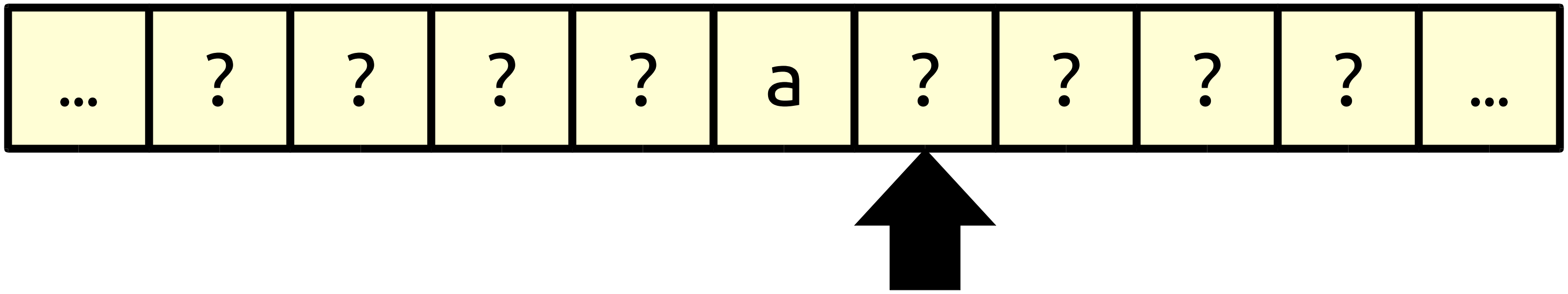

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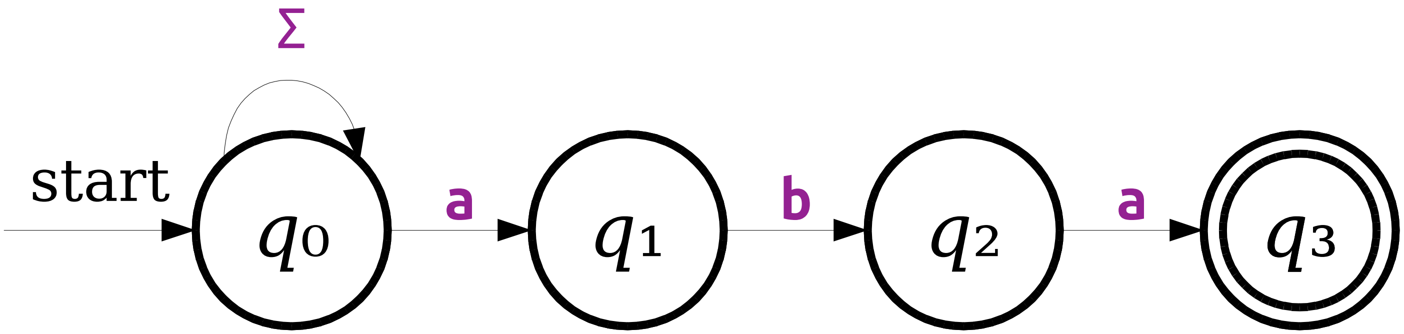

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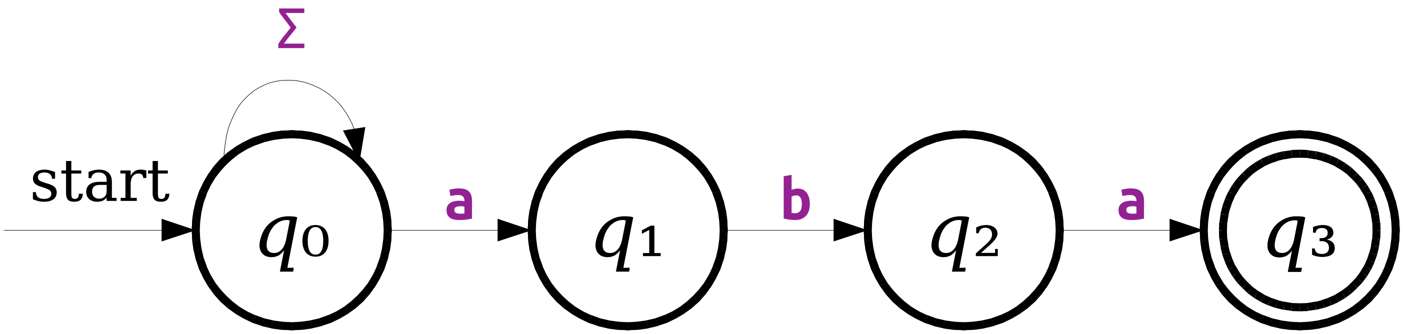

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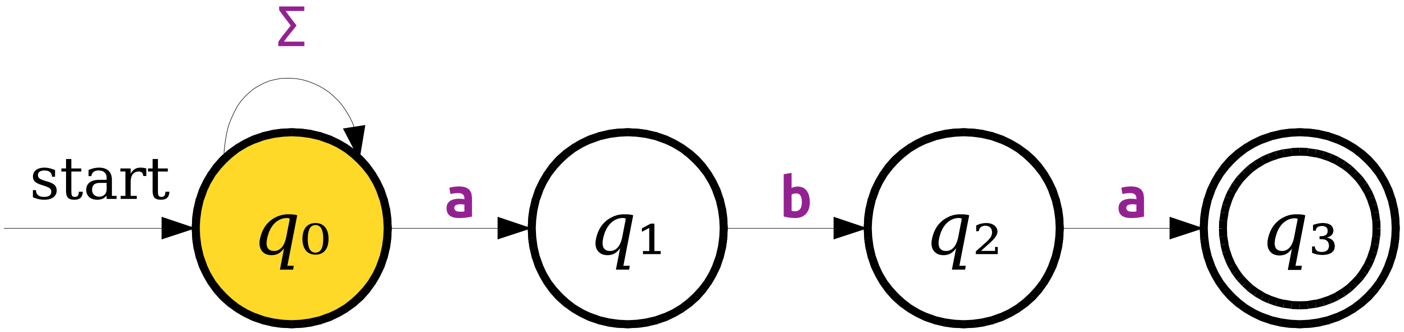

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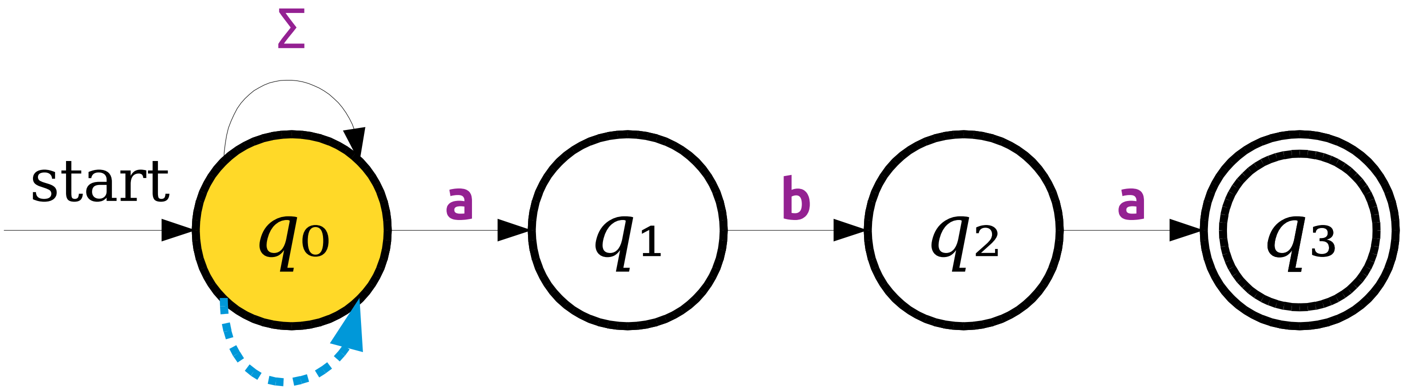

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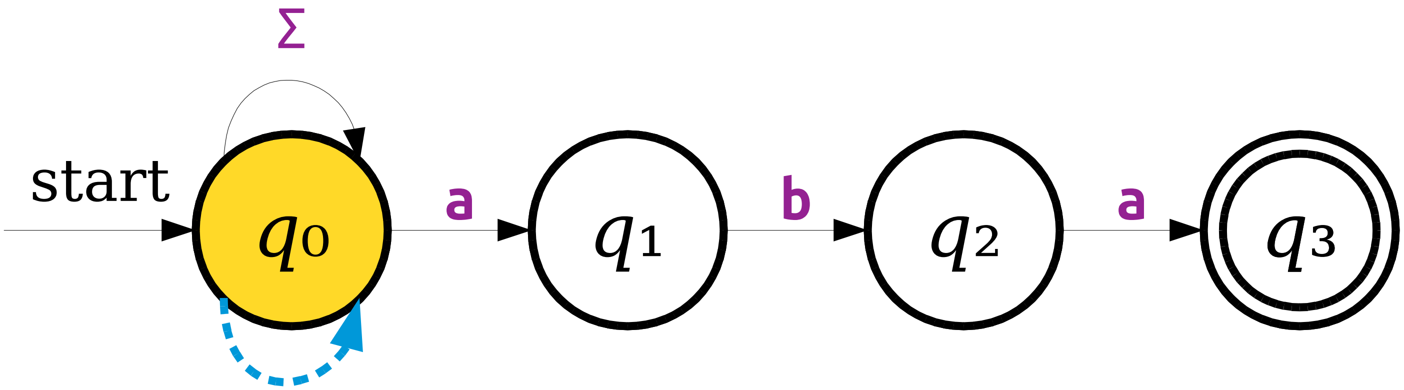

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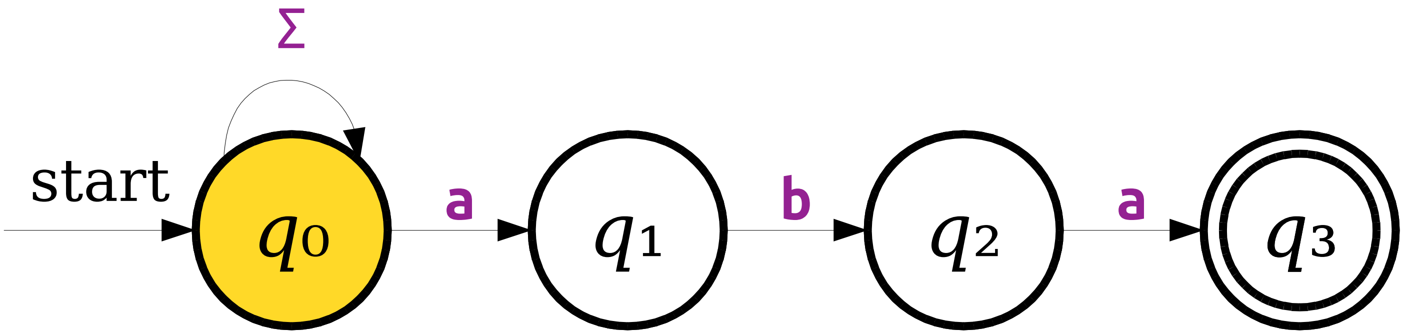

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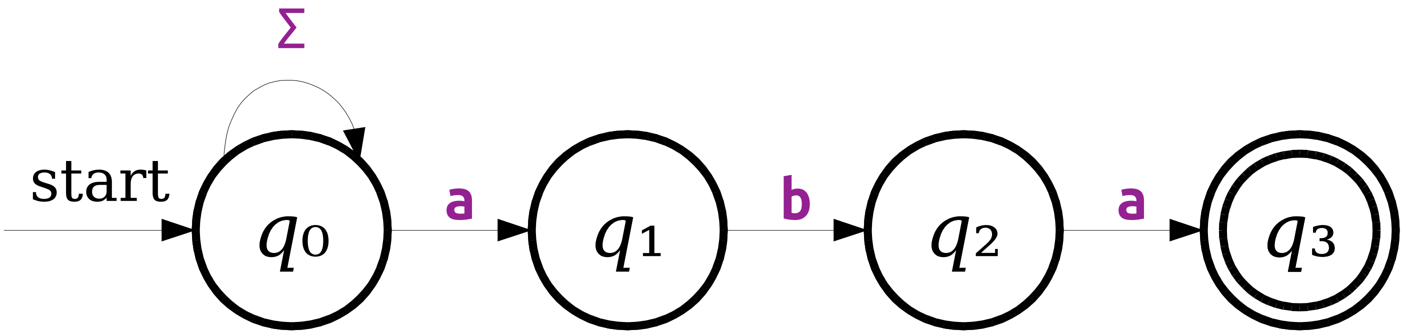

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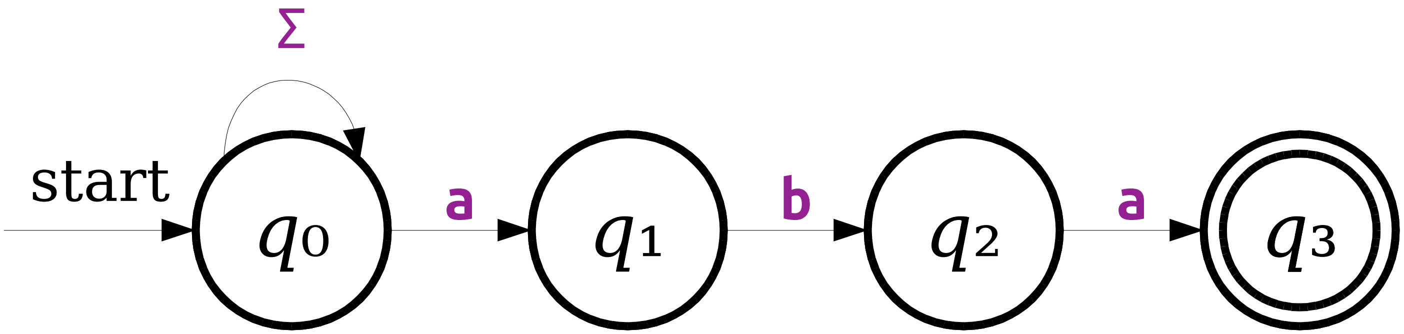

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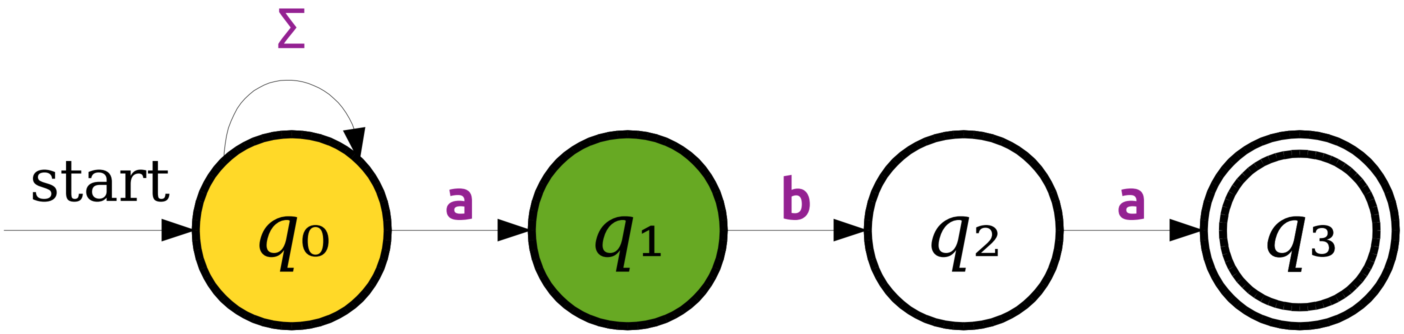

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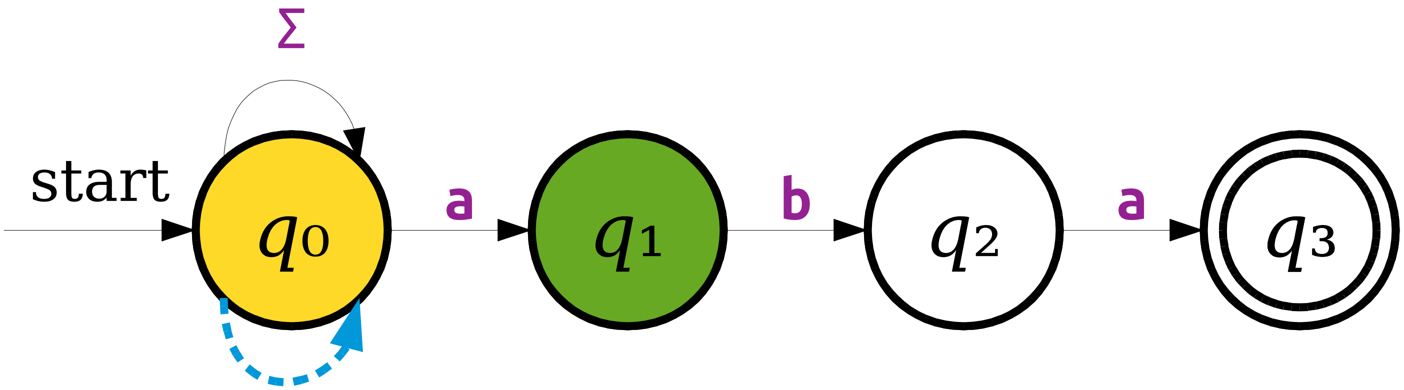

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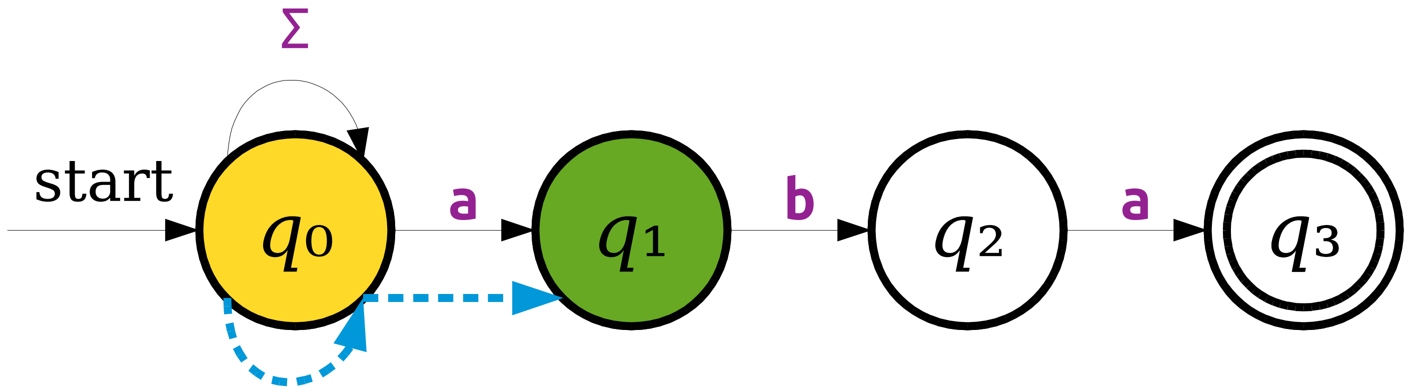

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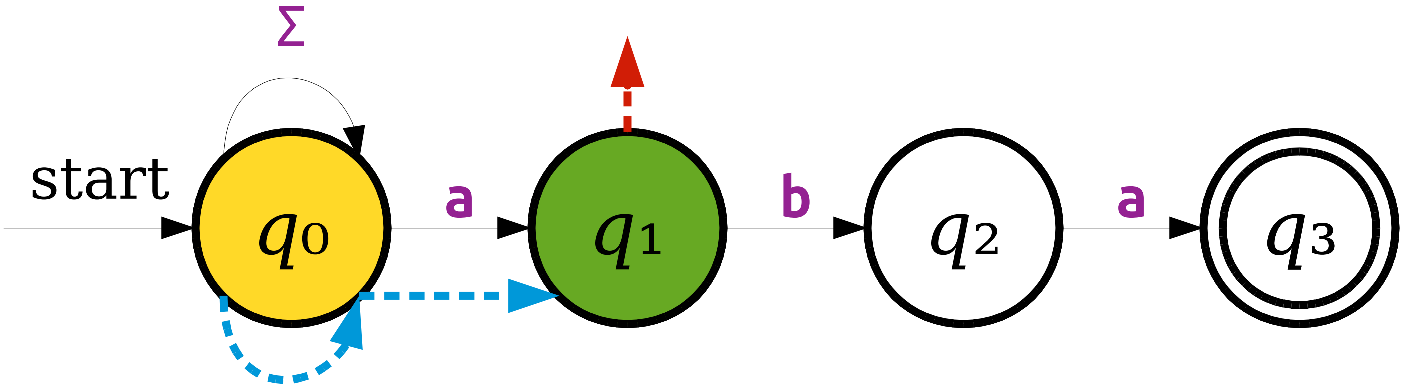

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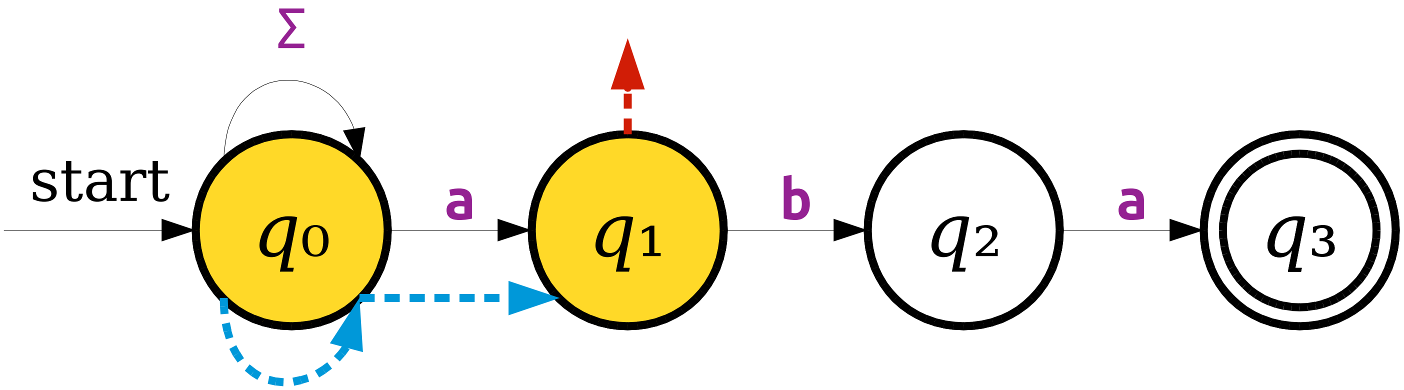

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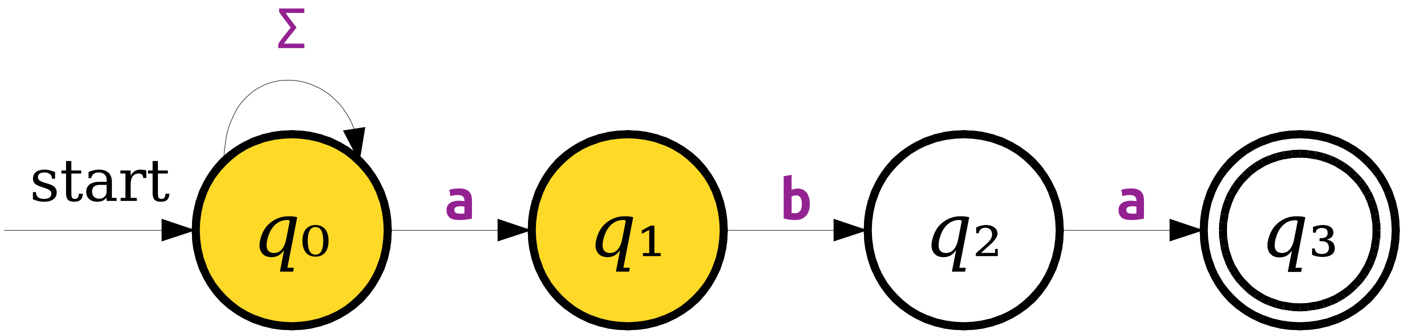

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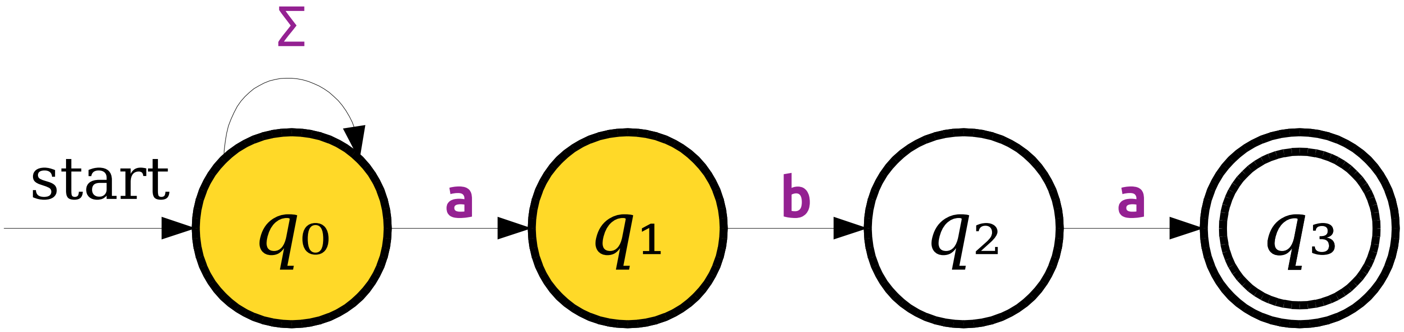

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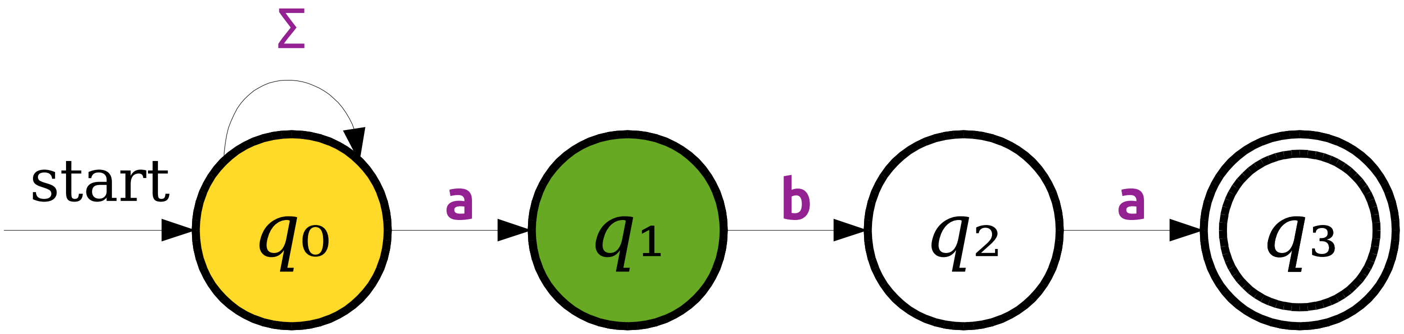

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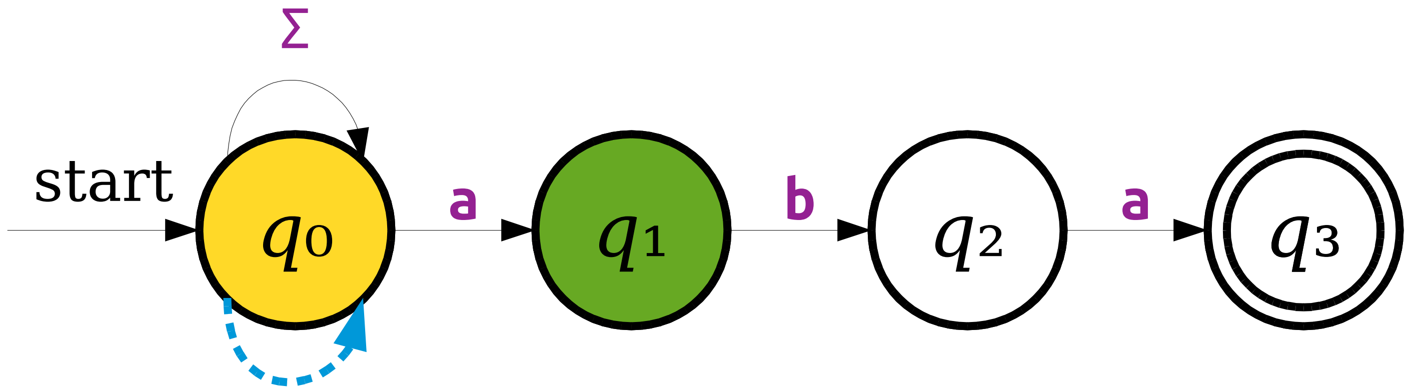

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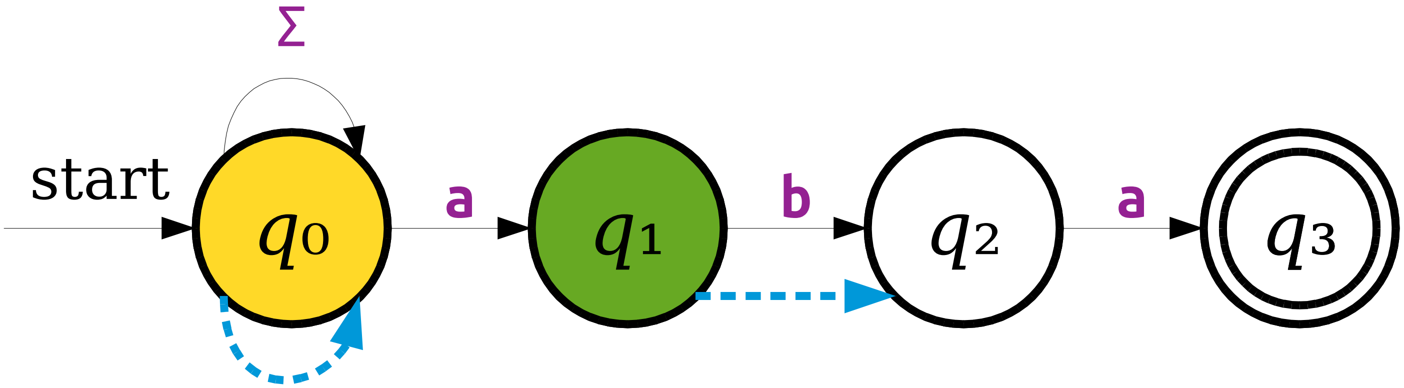

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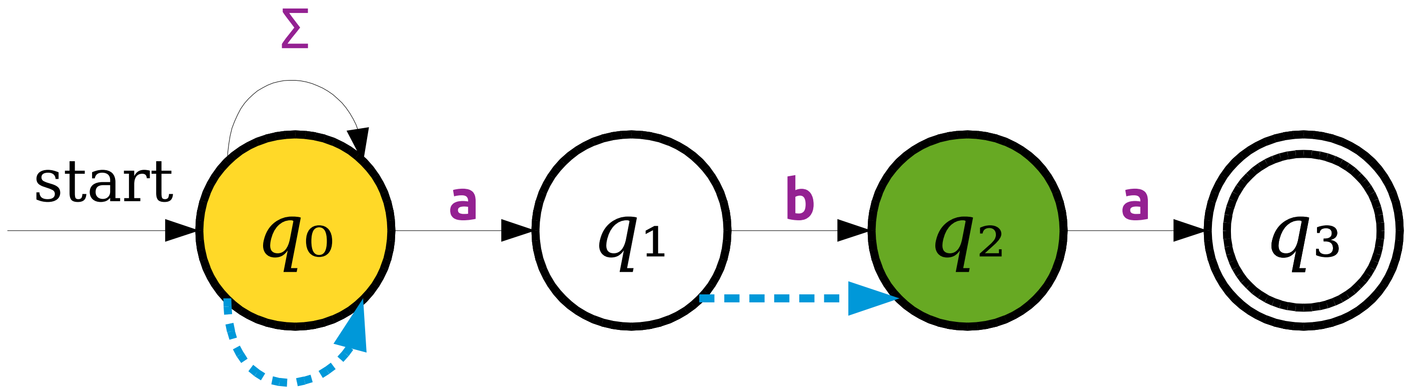

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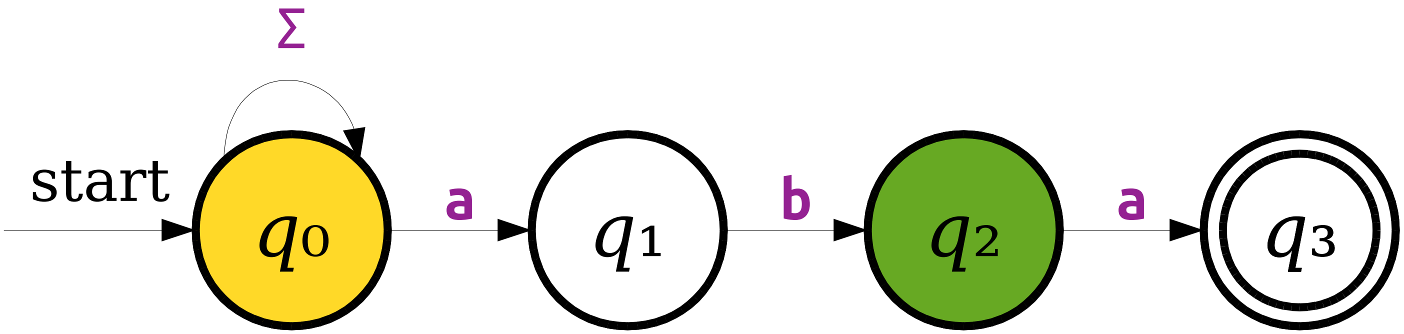

	a	b
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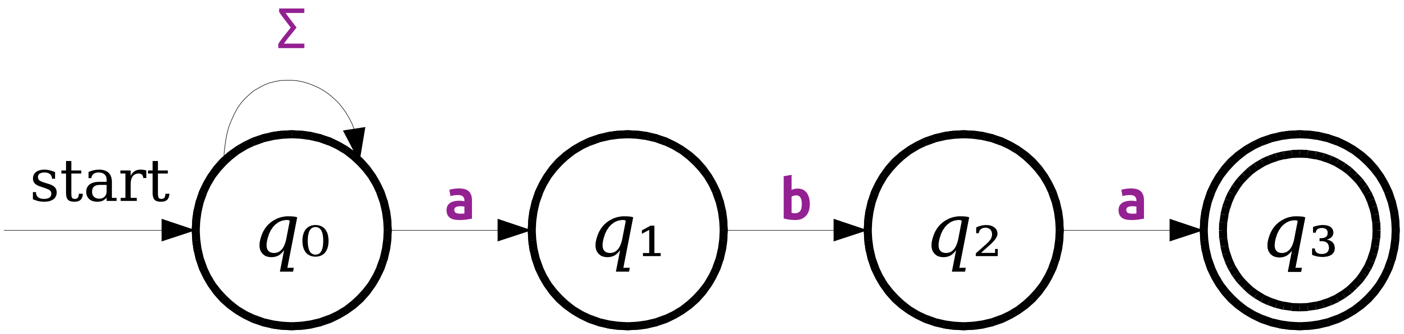

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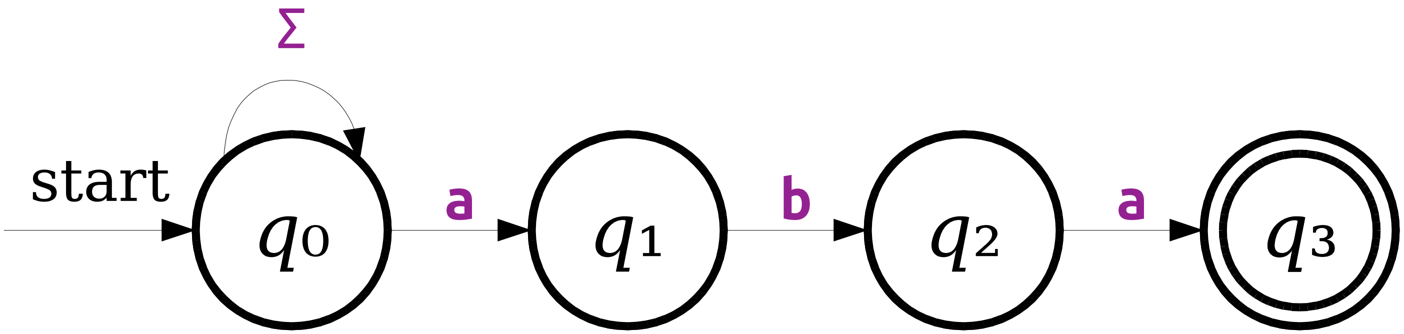

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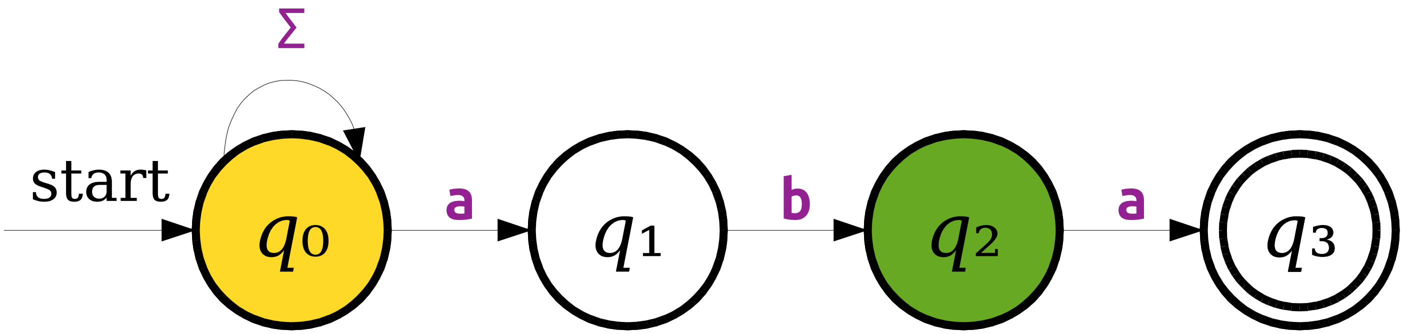

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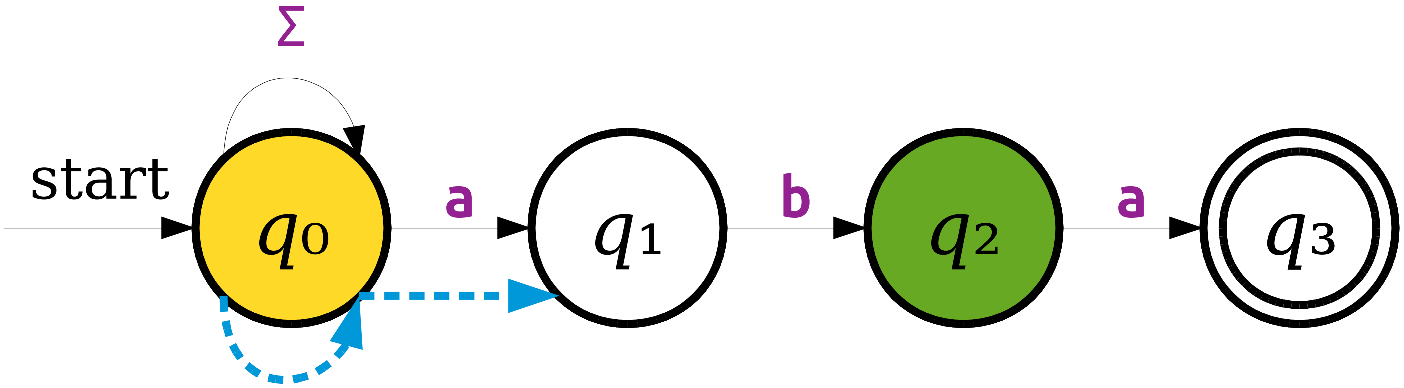

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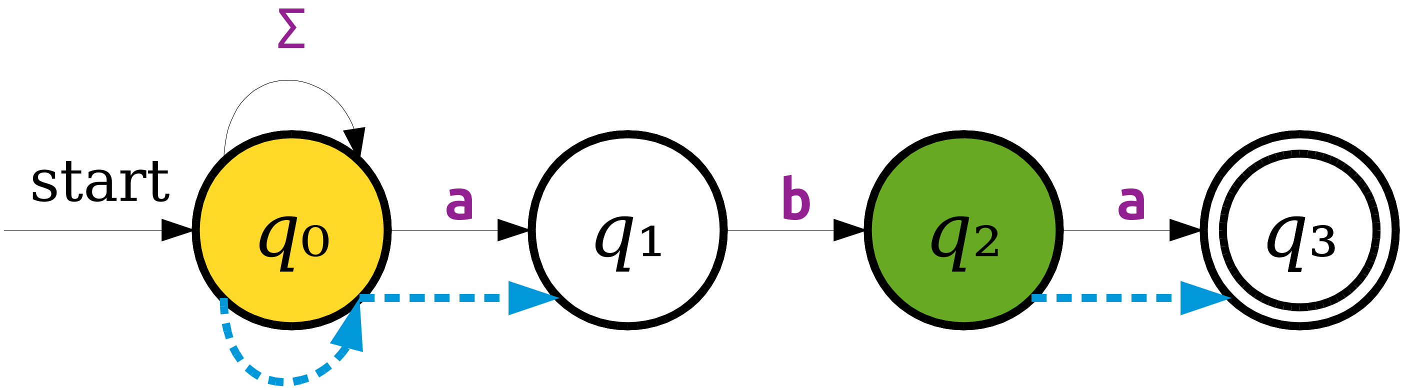

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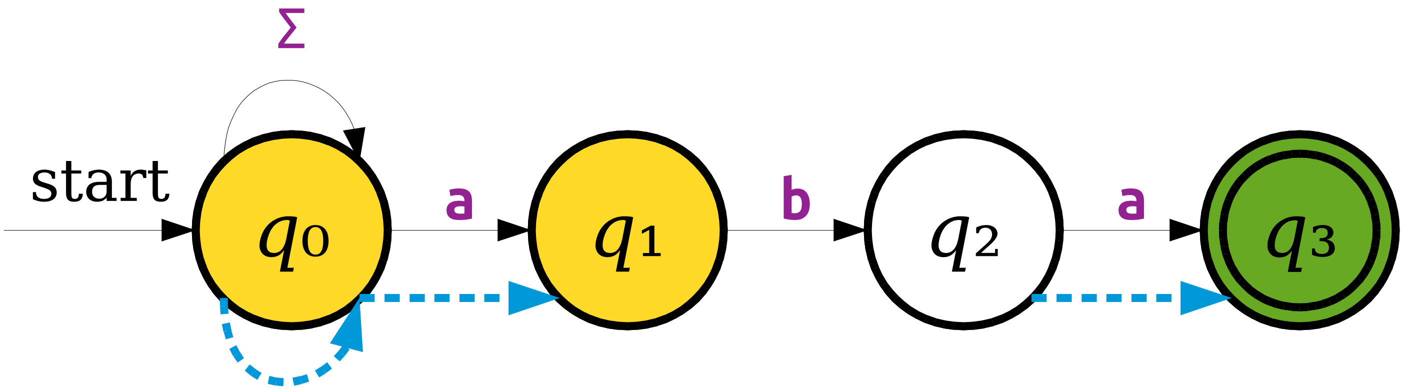

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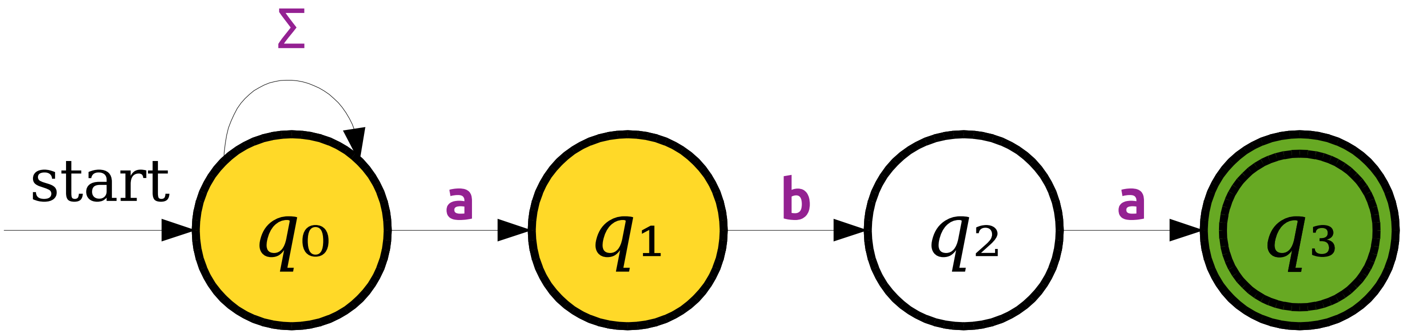

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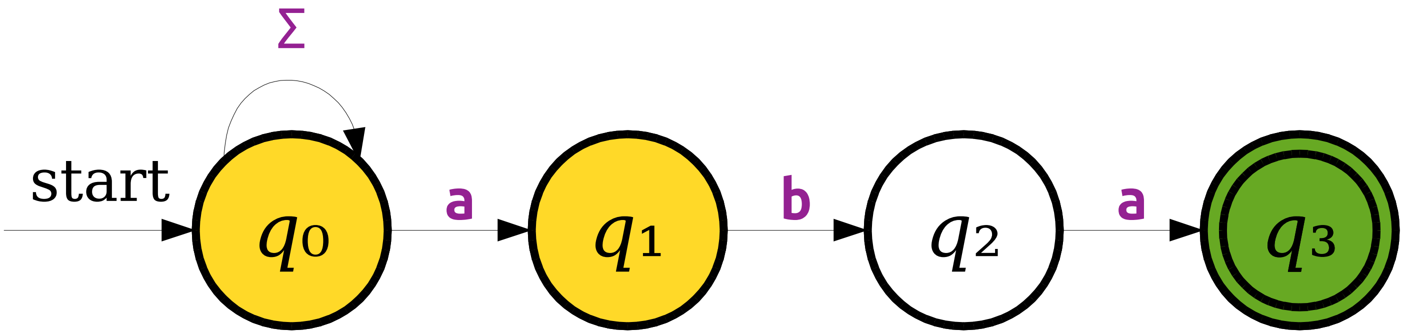

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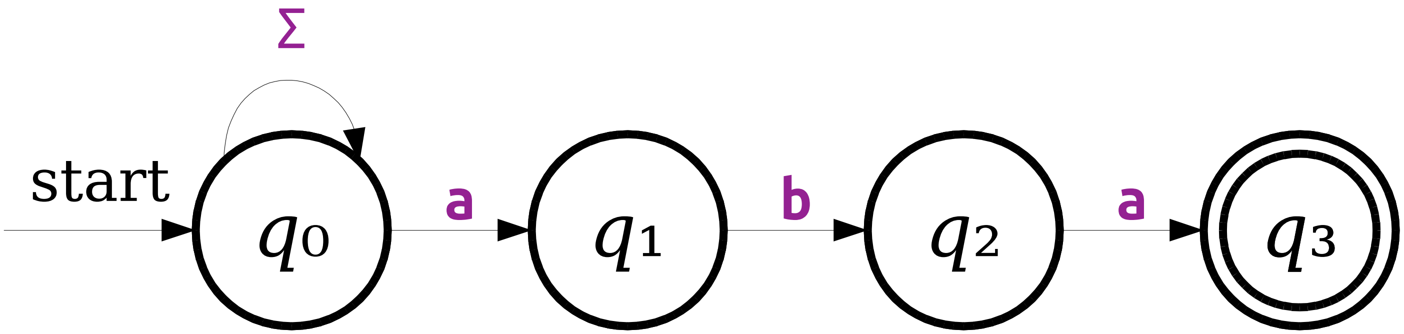

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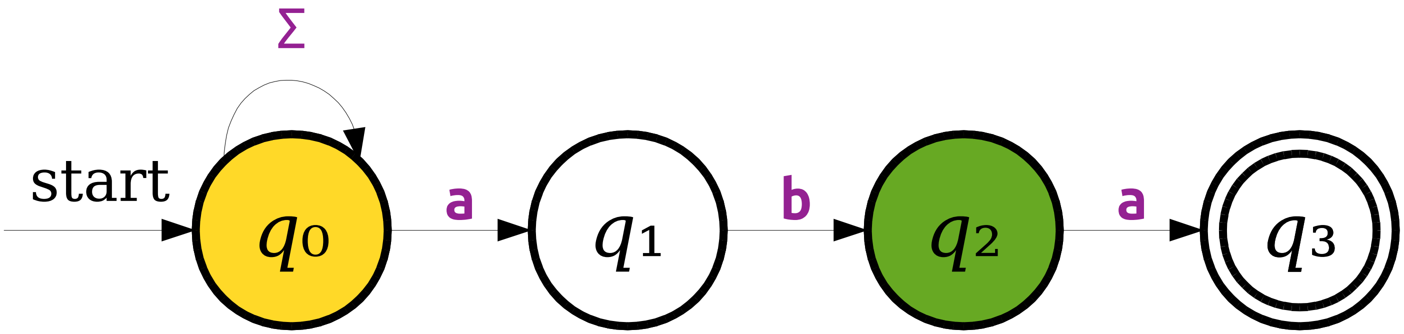

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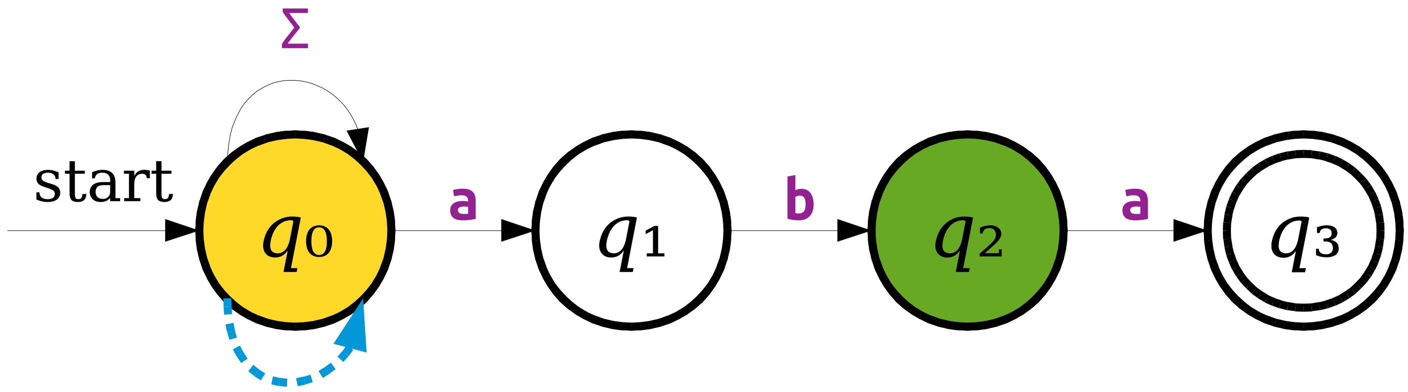

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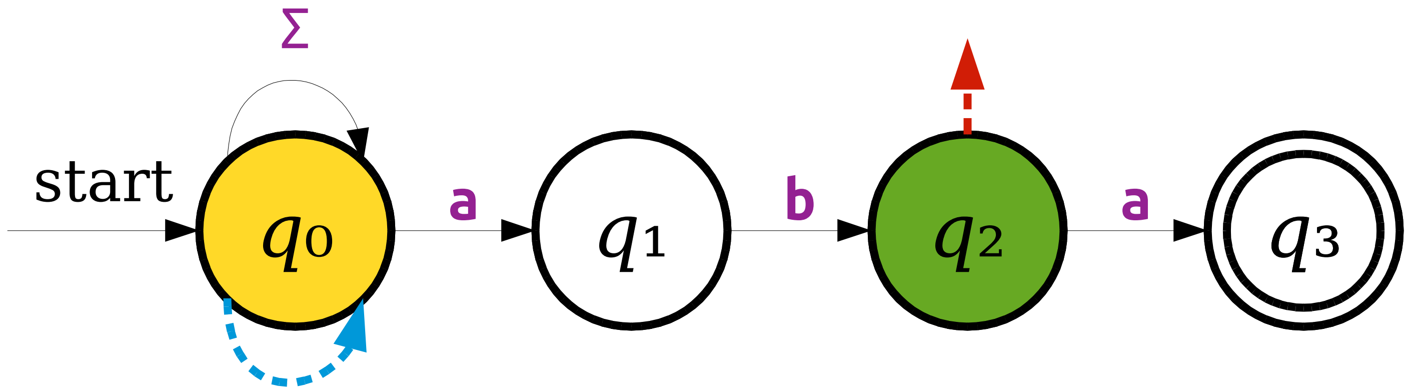

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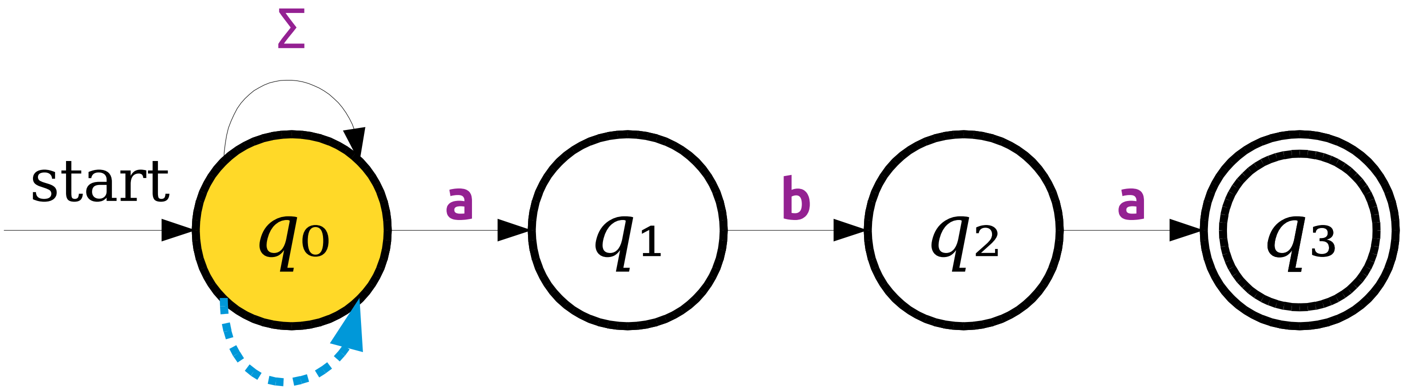

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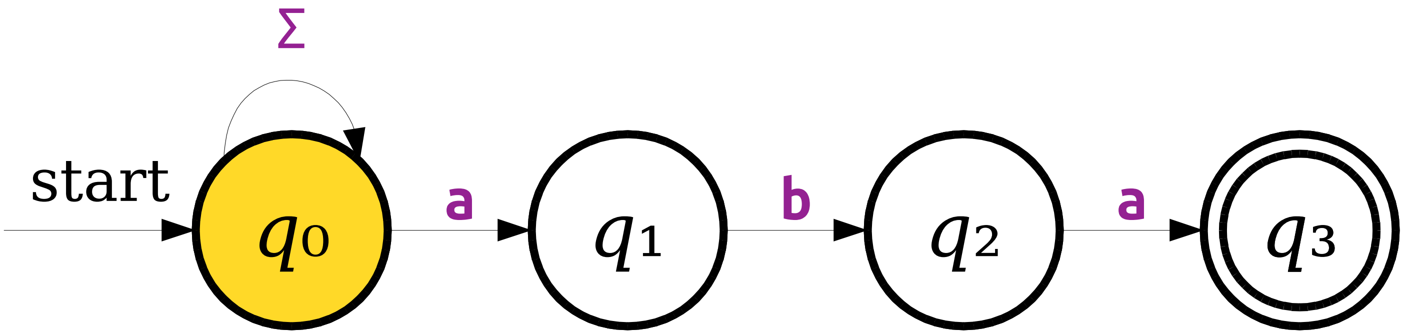

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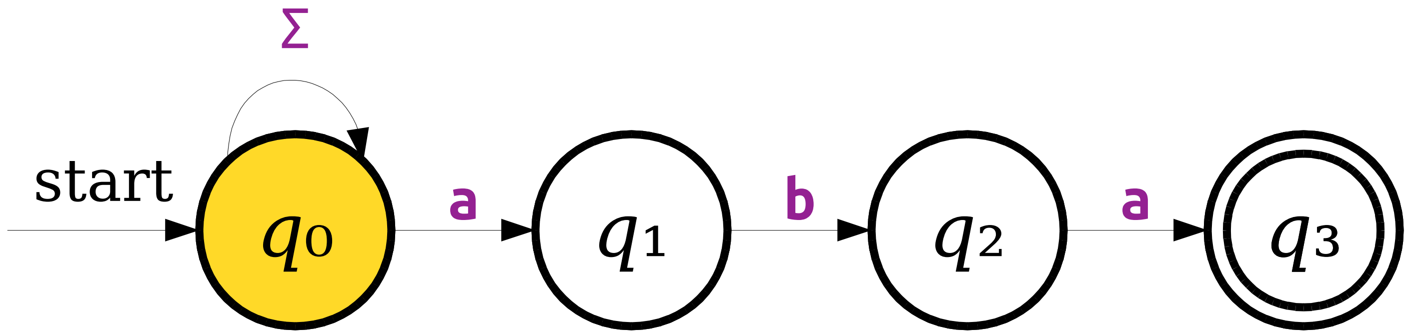

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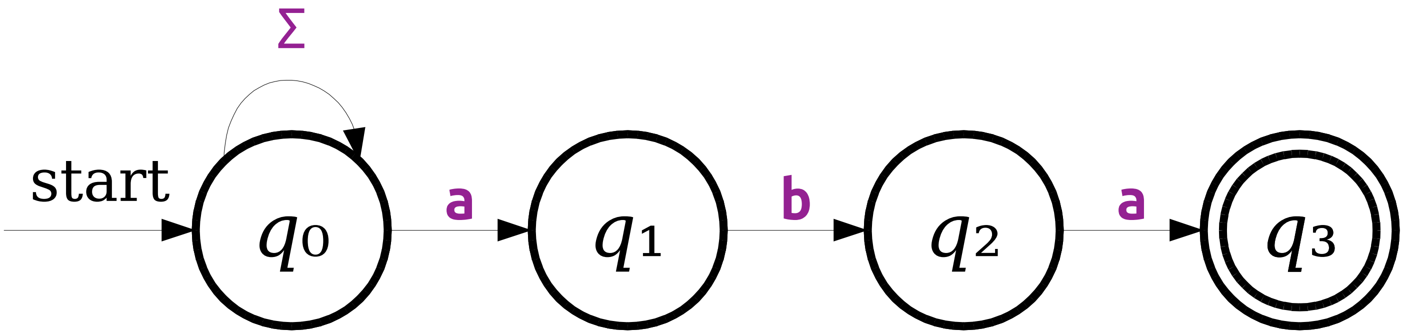

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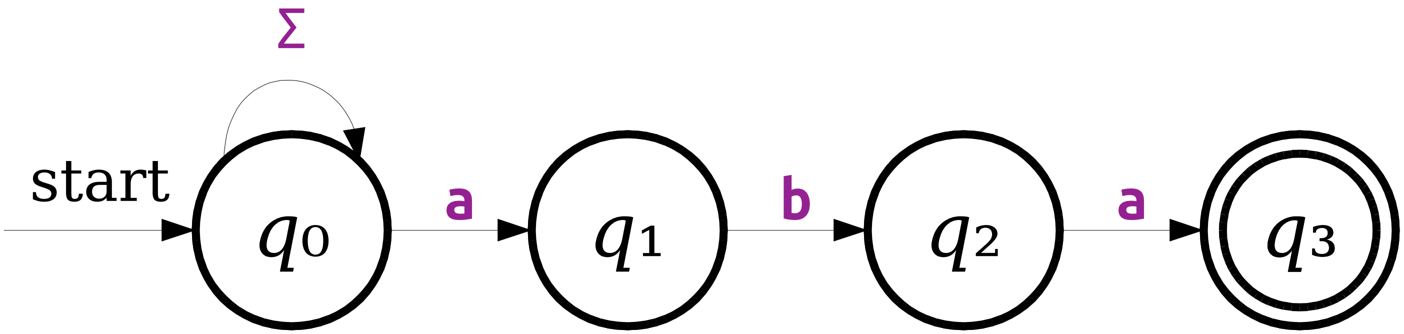

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$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
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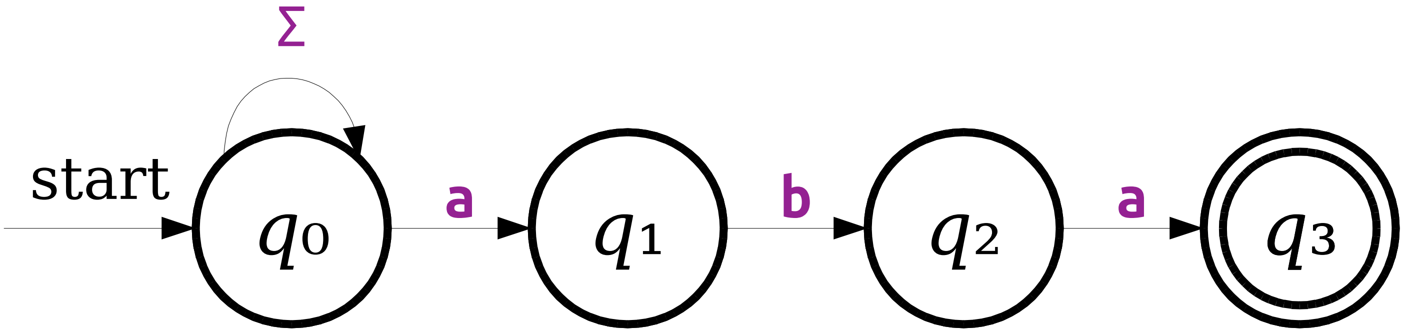
Fill in this row.

Answer at **https://cs103.stanford.edu/pollev**



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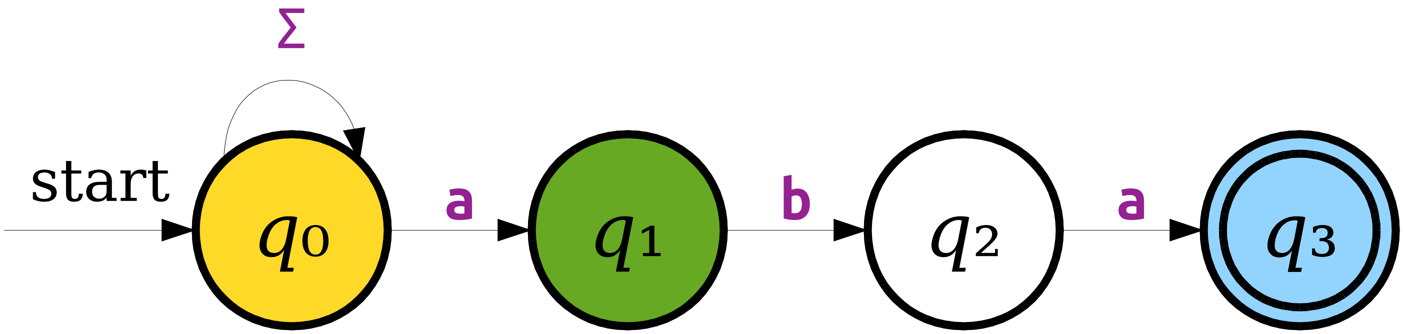

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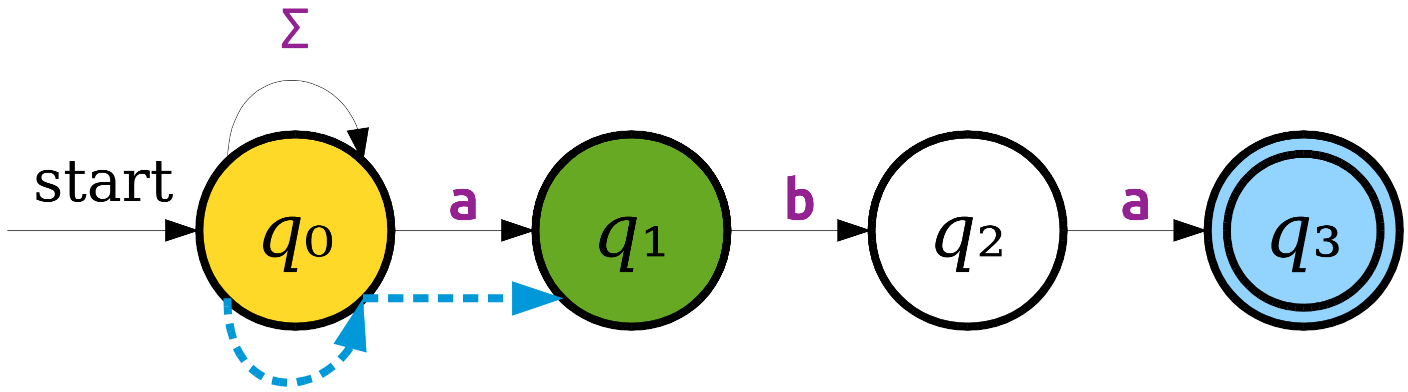

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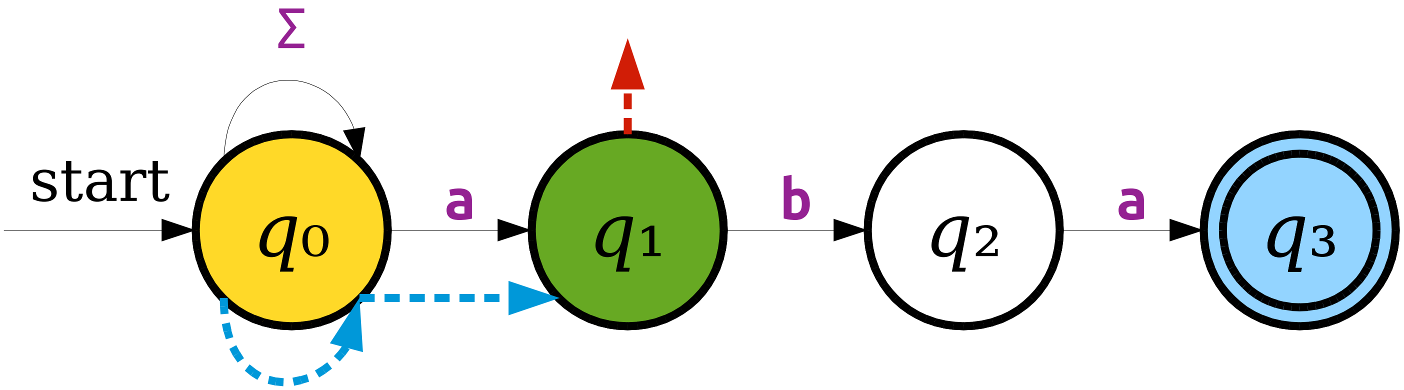

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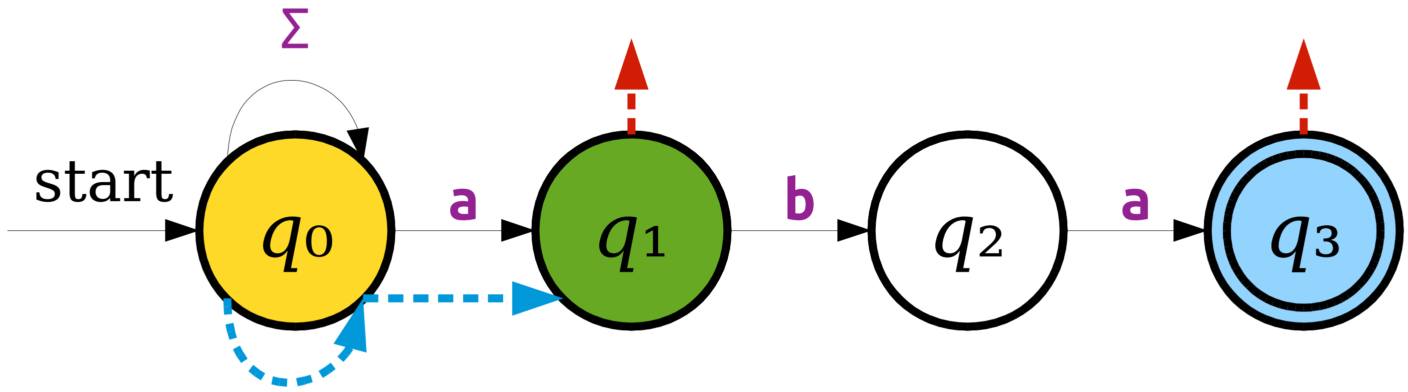

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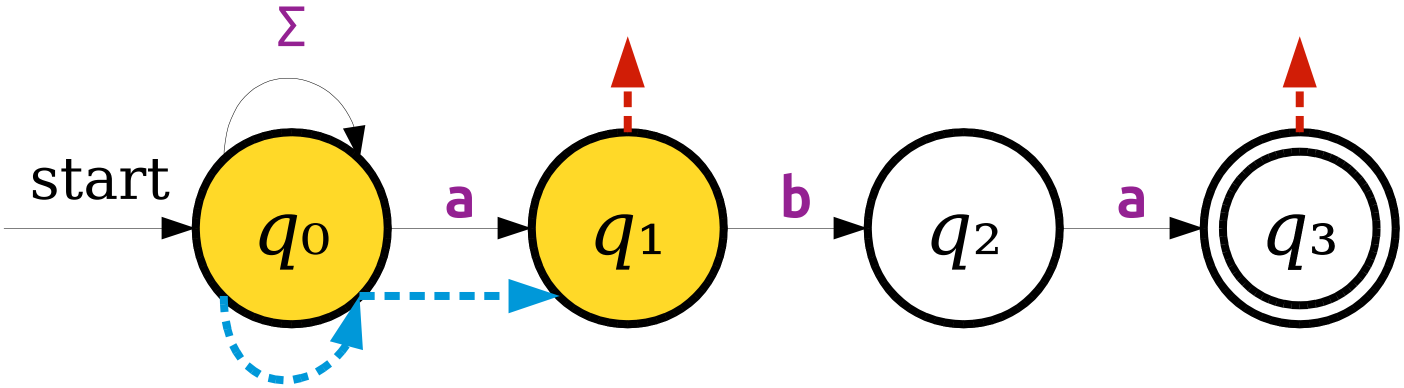

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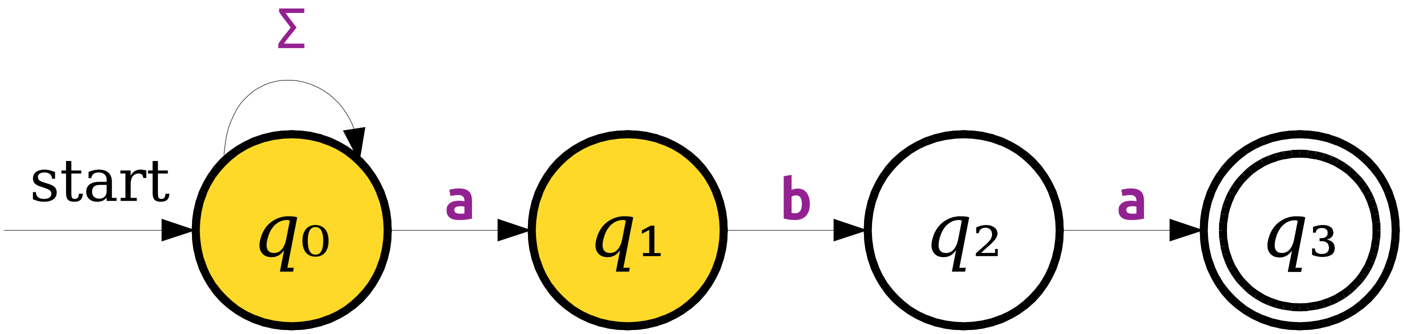

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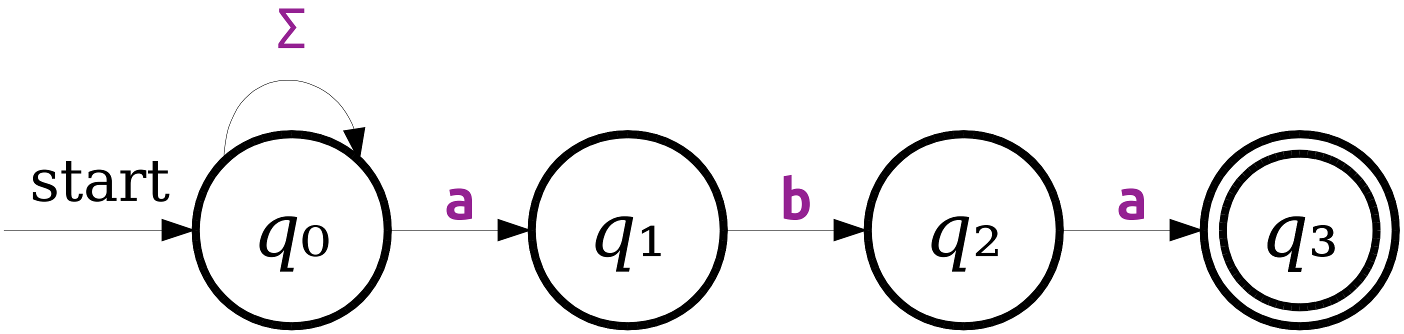

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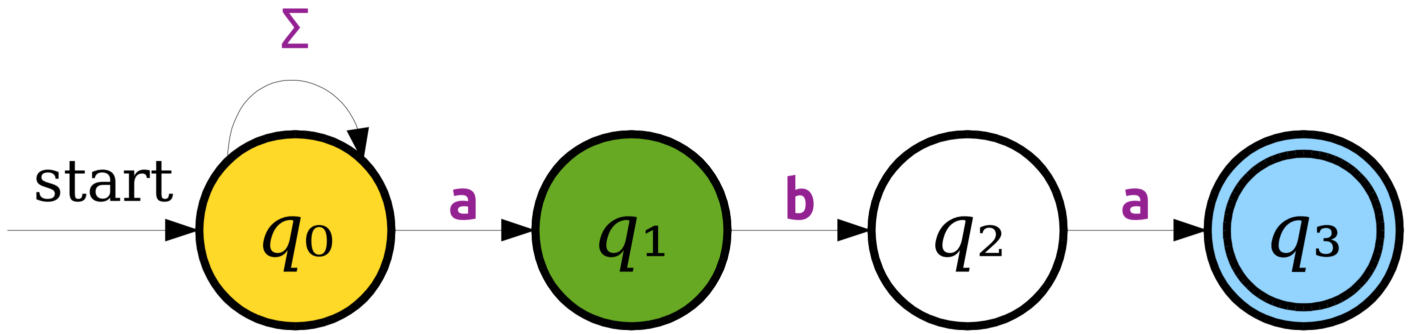

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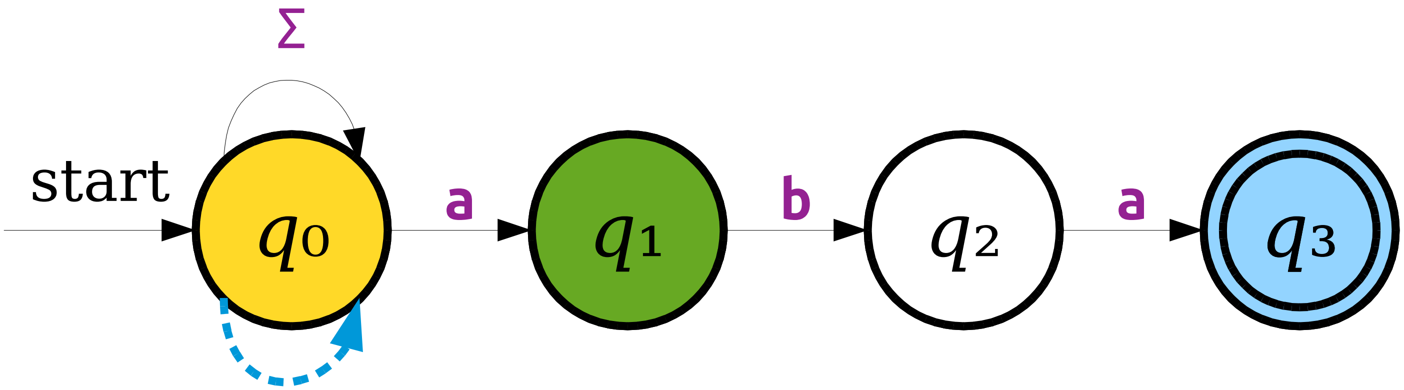

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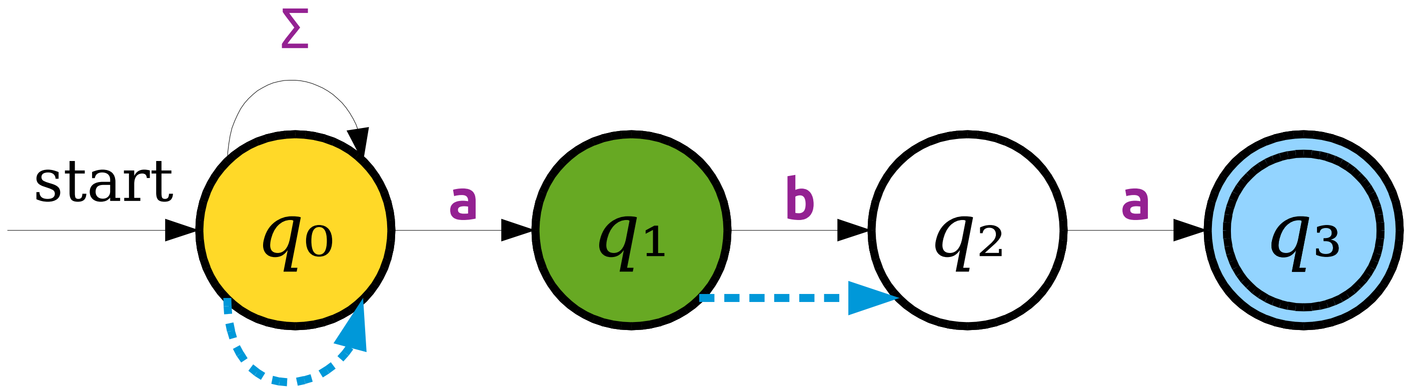

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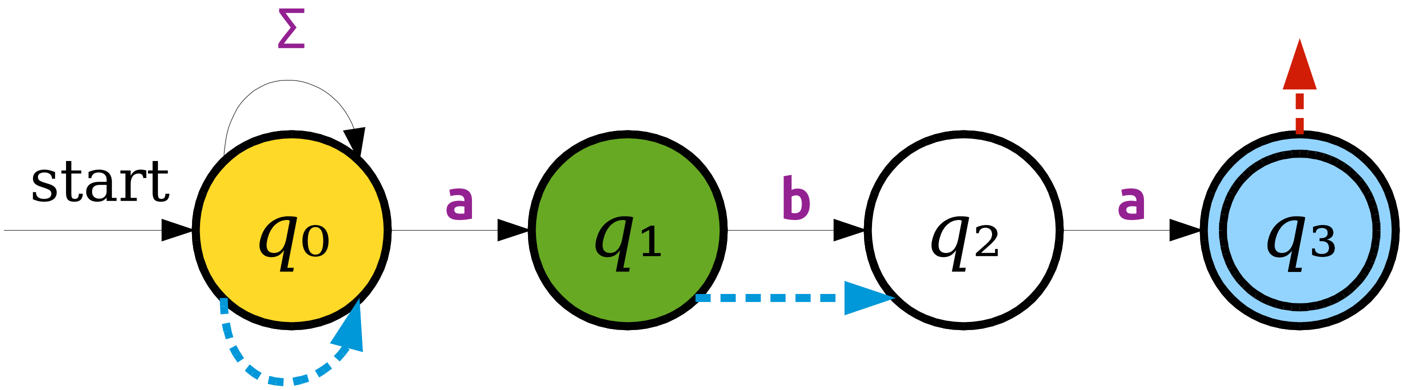

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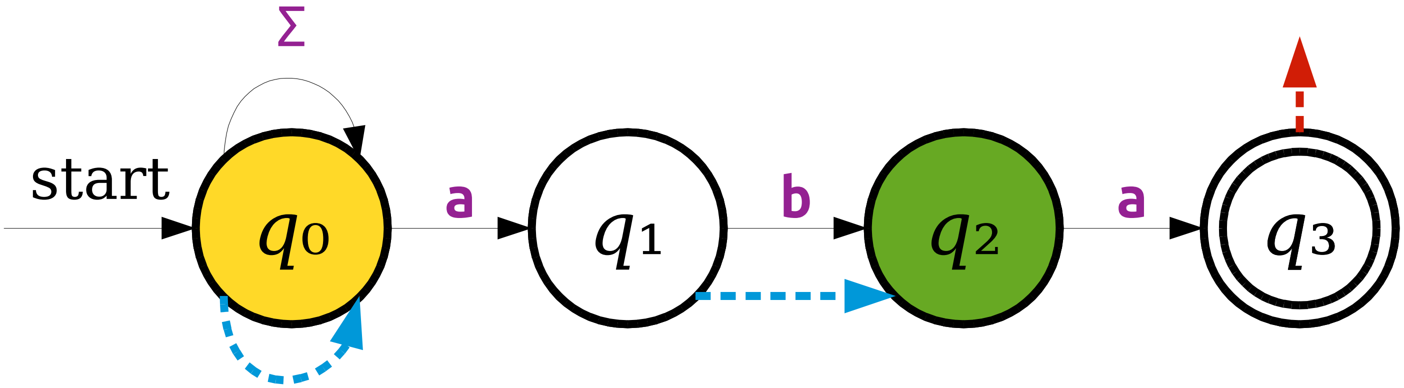

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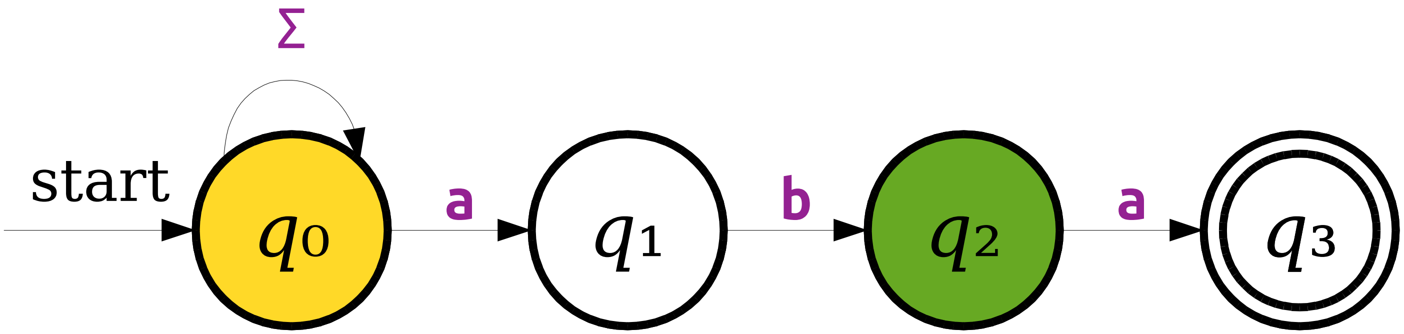

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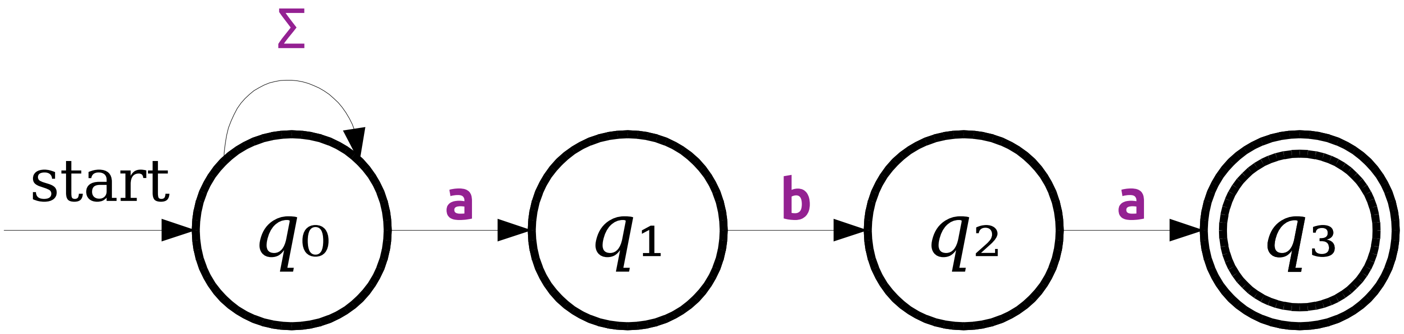

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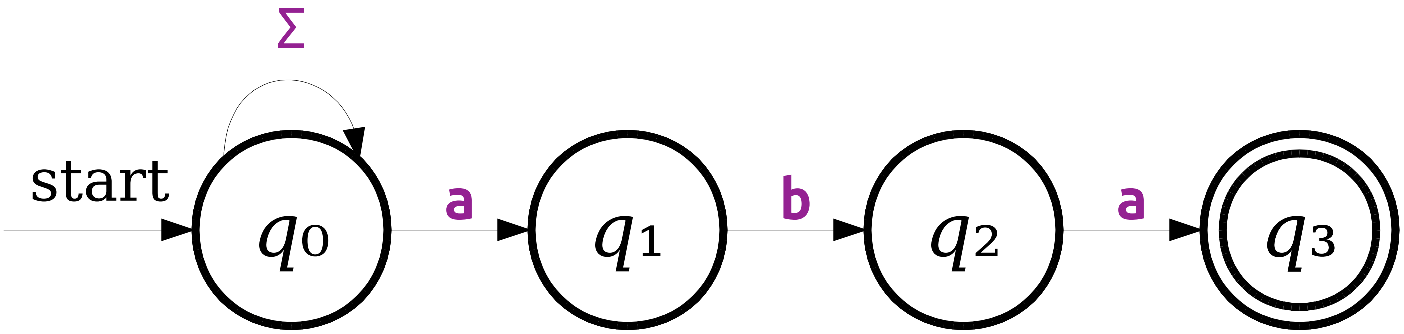

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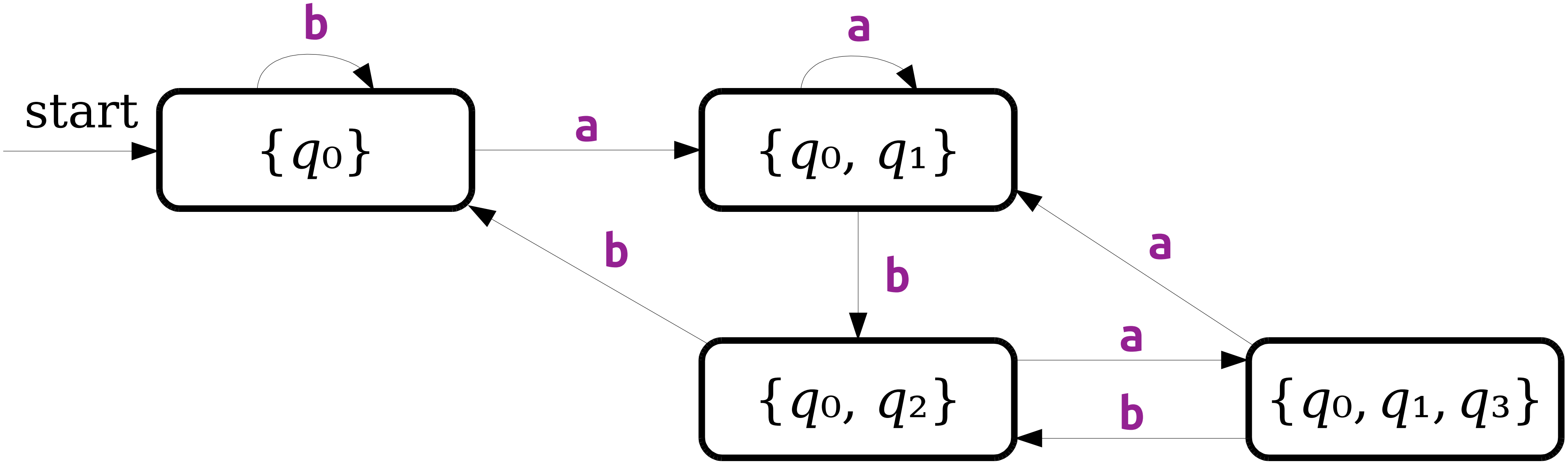

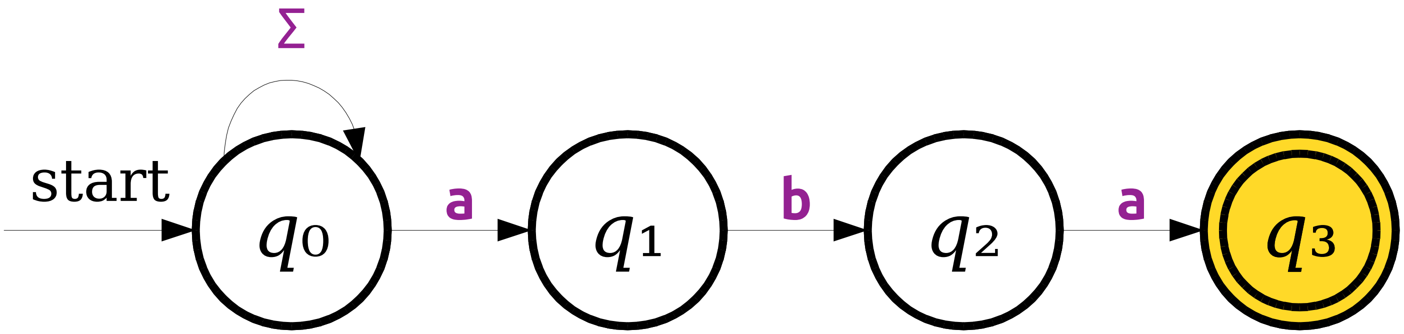

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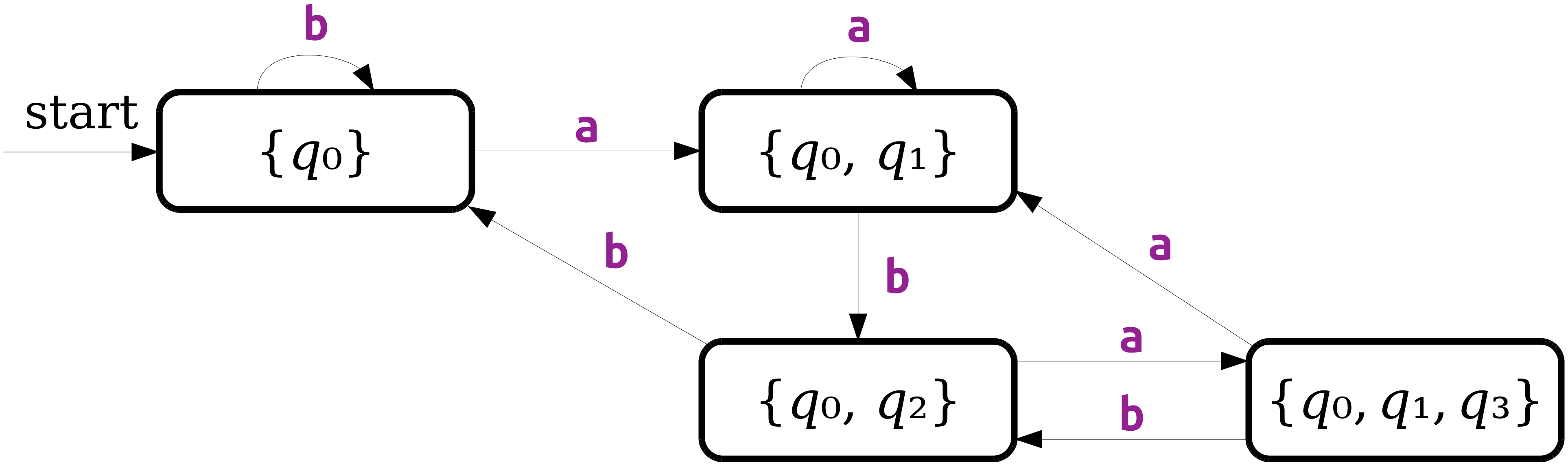

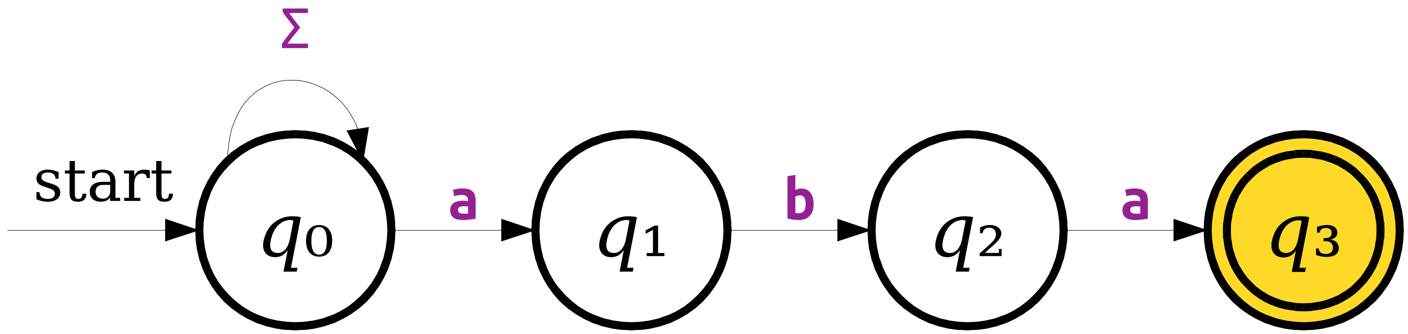

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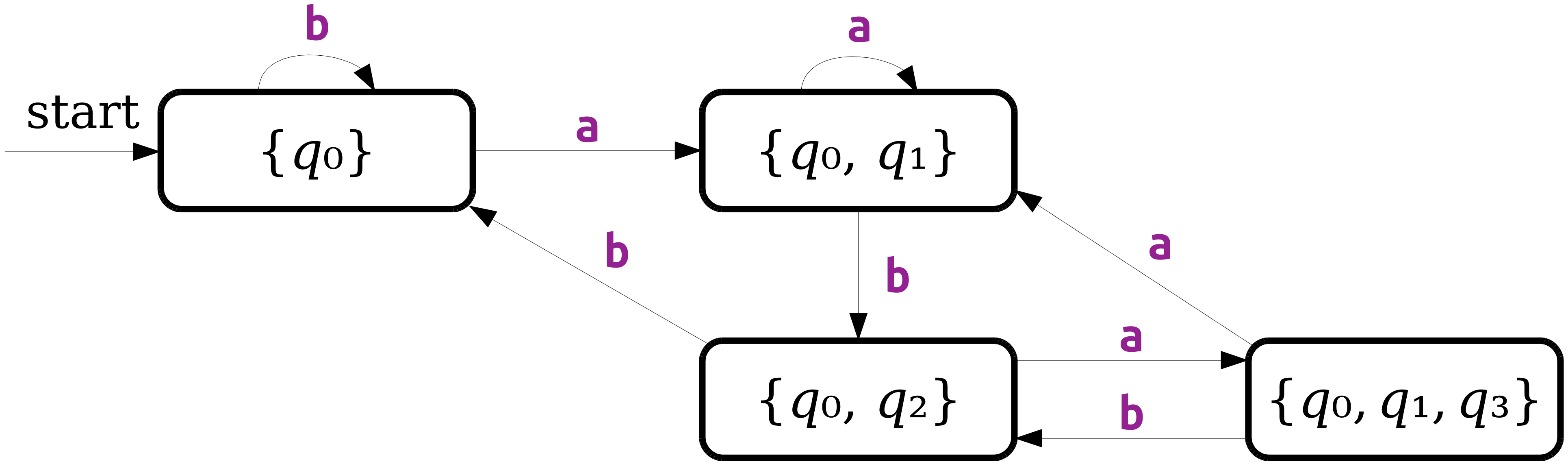
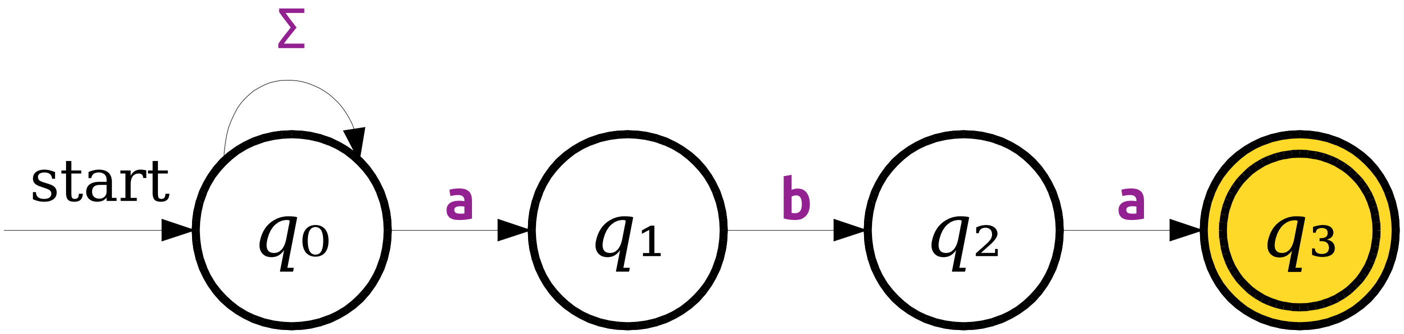
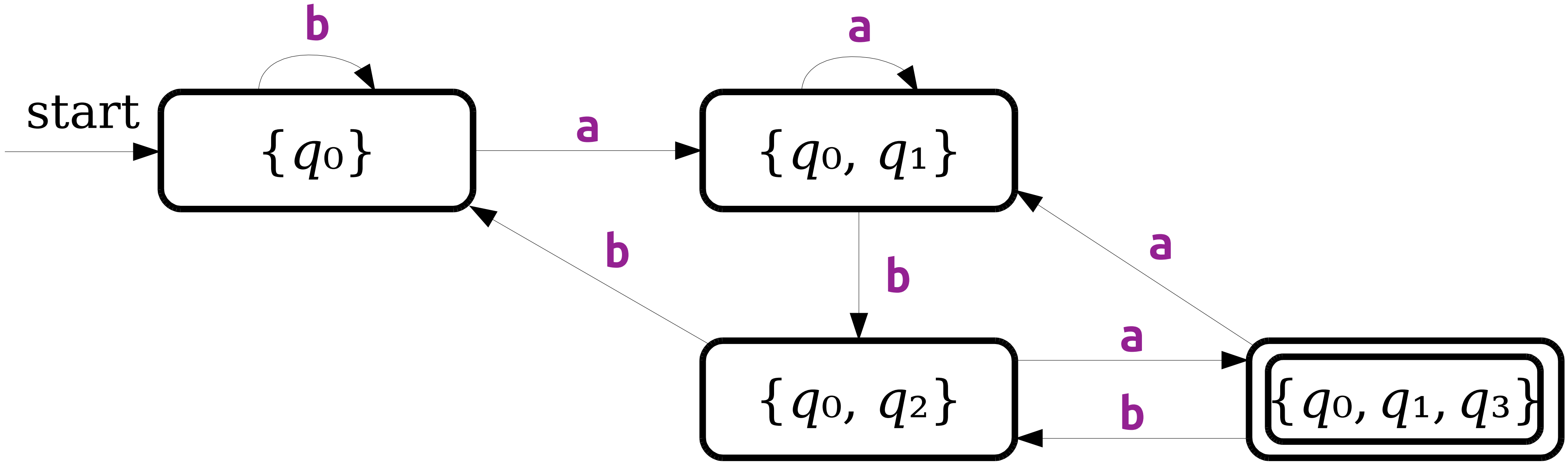
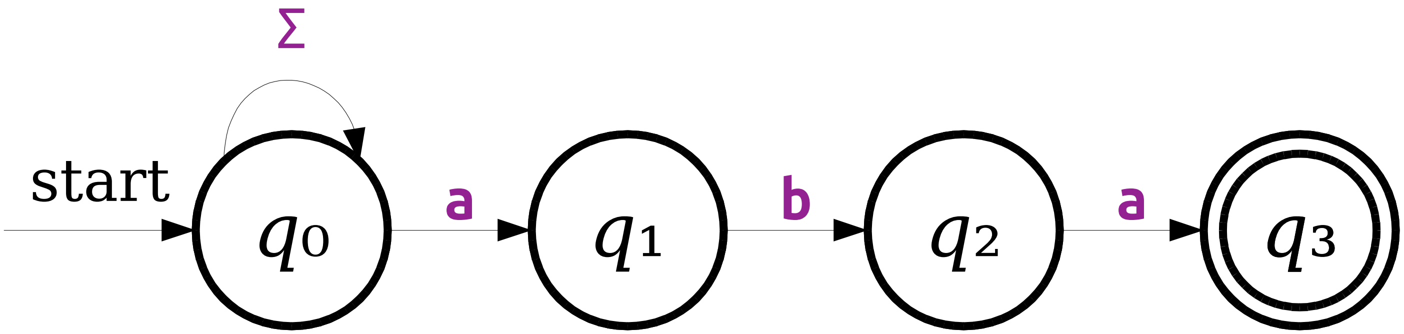


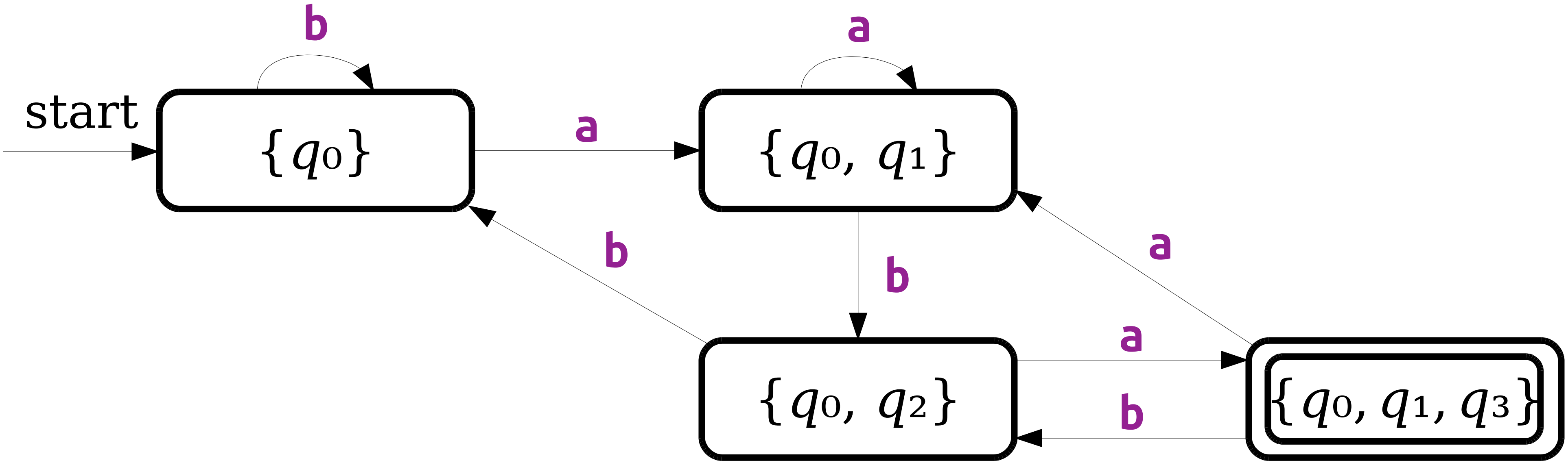


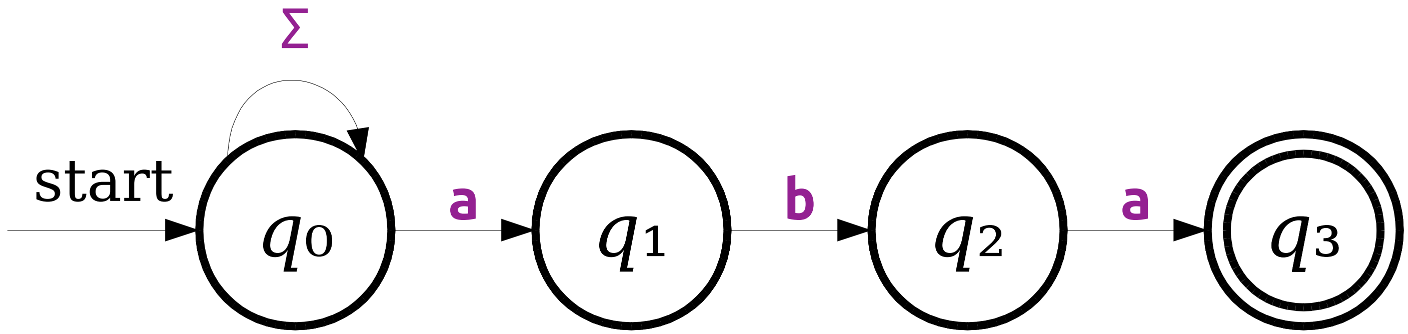
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$*\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$



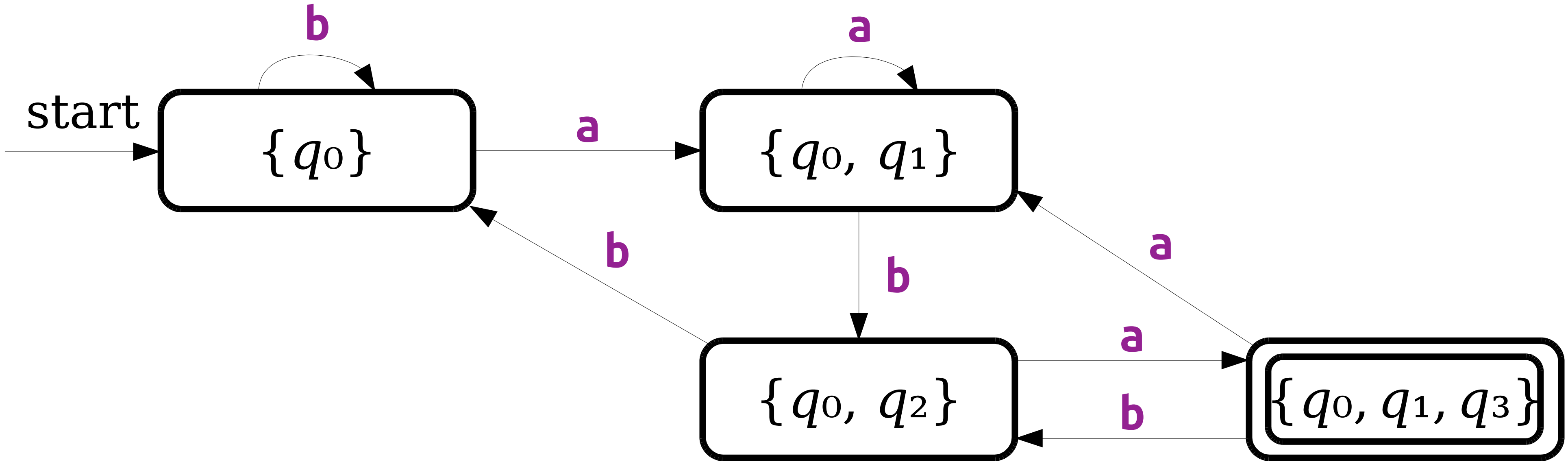


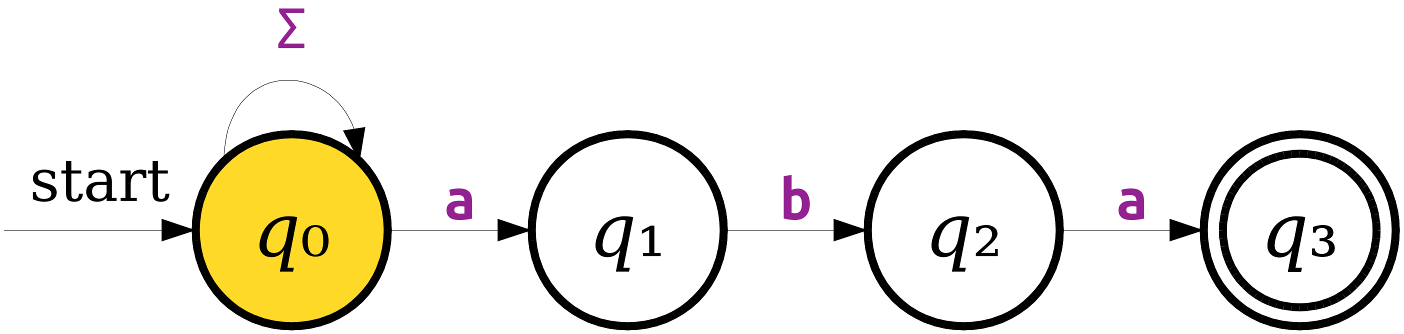
	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$*\{q_0, q_1, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$



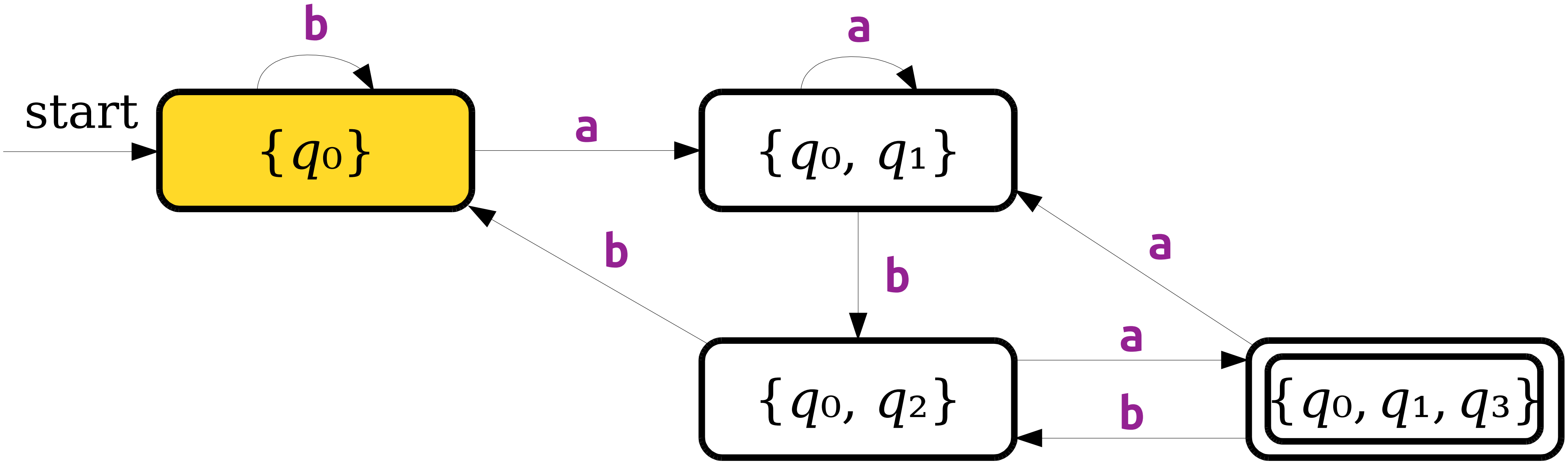


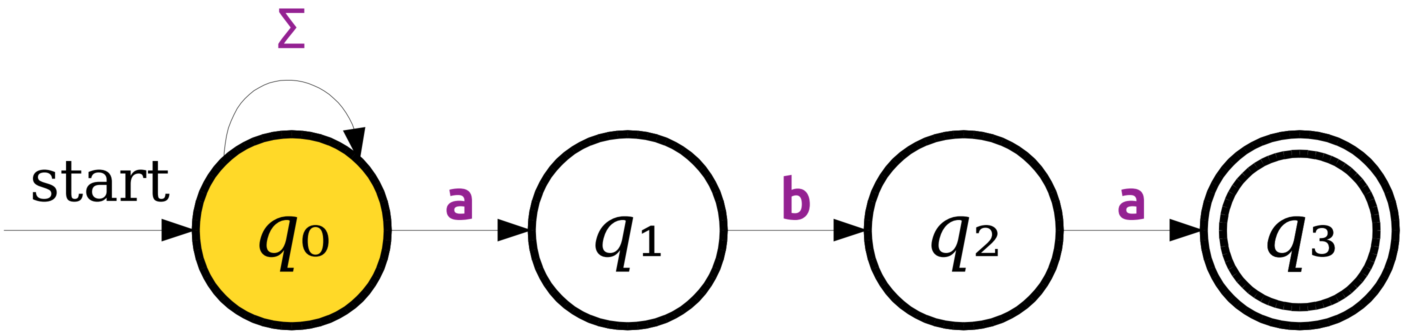
a	b	a	a	b	a

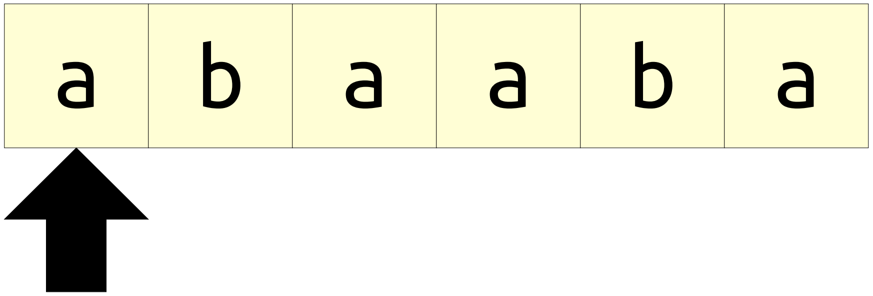


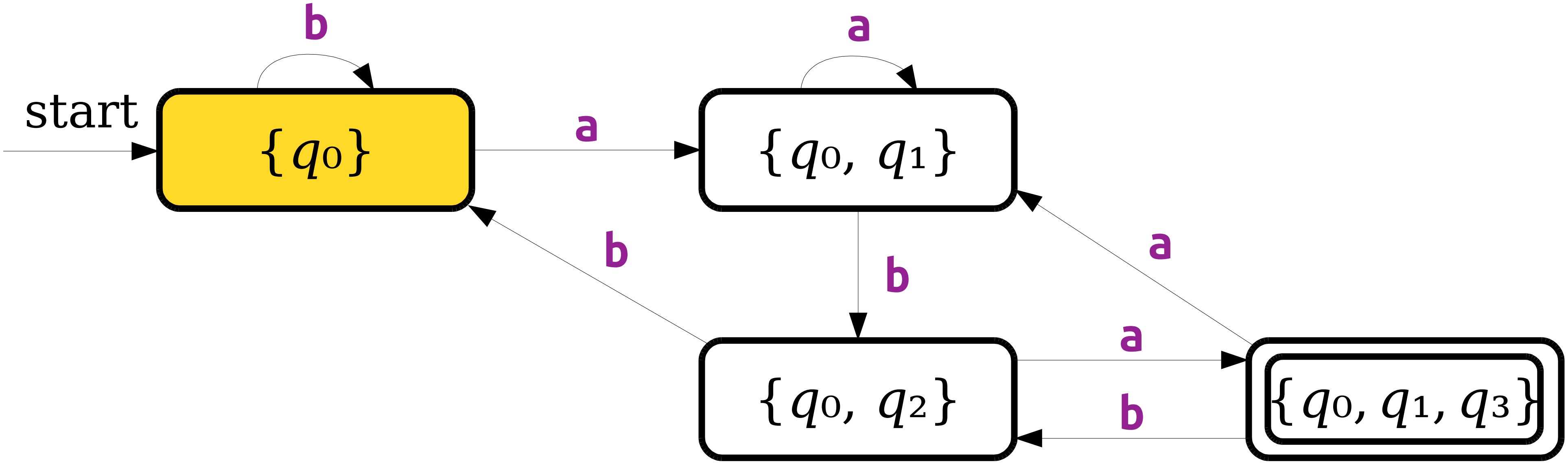


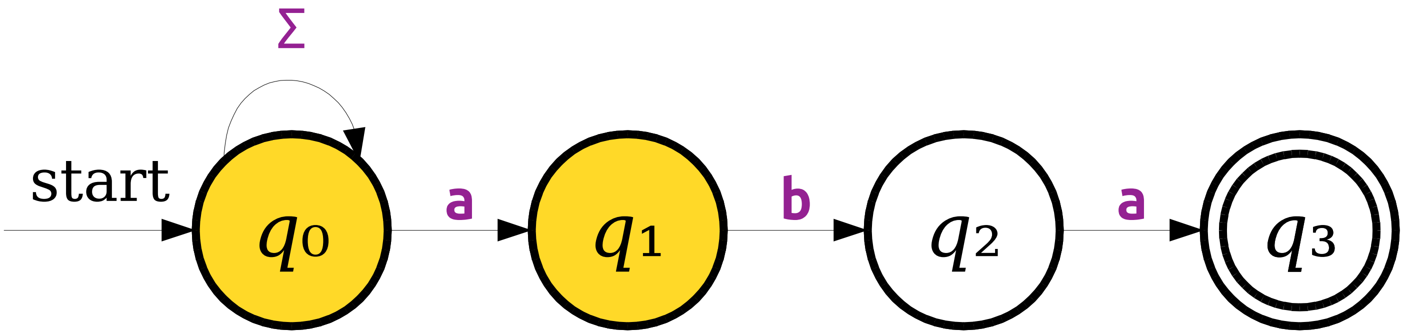
a	b	a	a	b	a

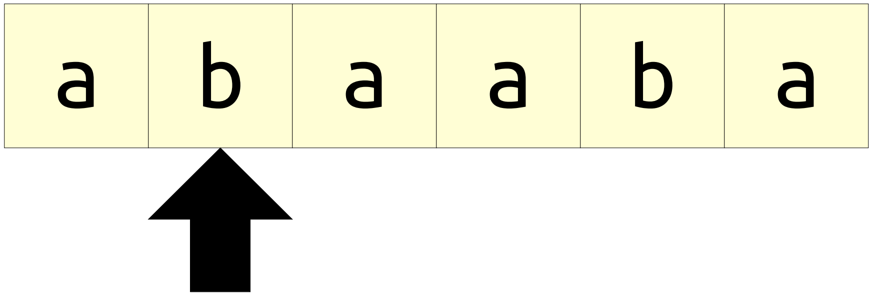



Σ
start
q_0
a
q_1
b
q_2
a
q_3

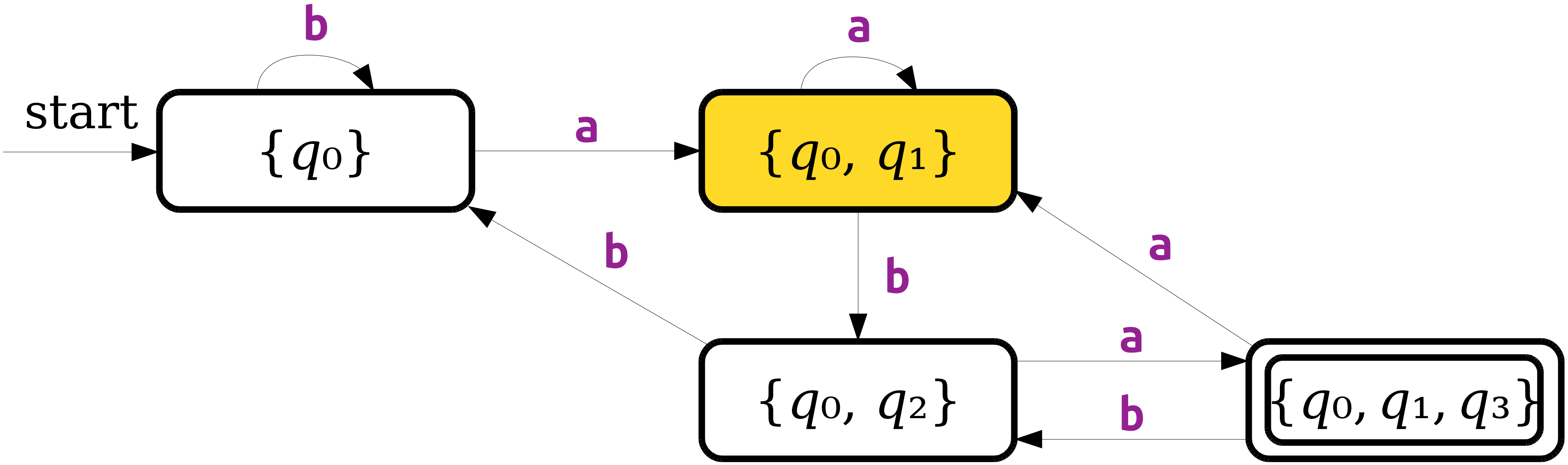

a
b
a
a
b
a

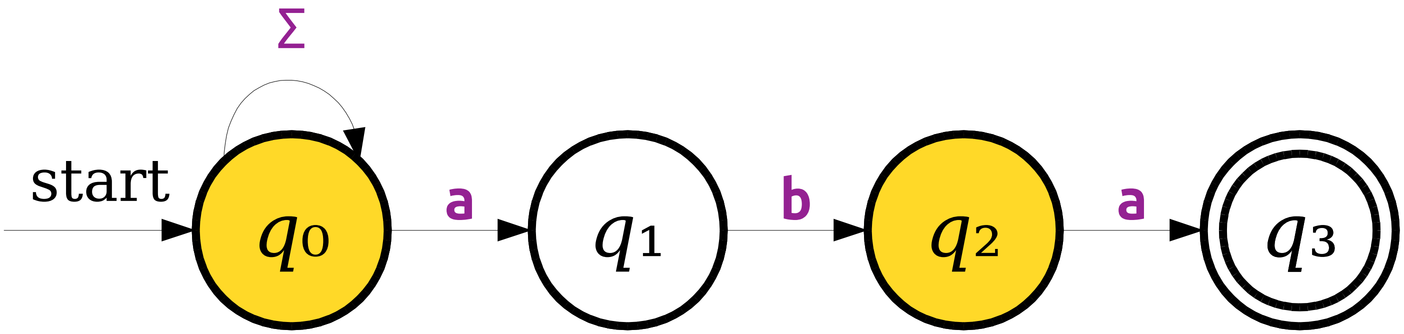

b
a
start
$\{q_0\}$
a
$\{q_0, q_1\}$
b
b
a
a
b
$\{q_0, q_2\}$
$\{q_0, q_1, q_3\}$

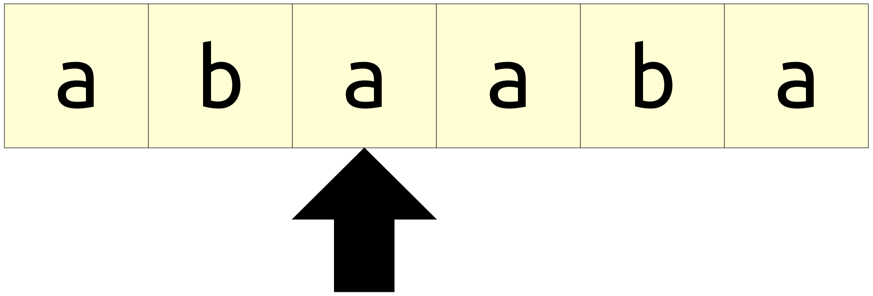


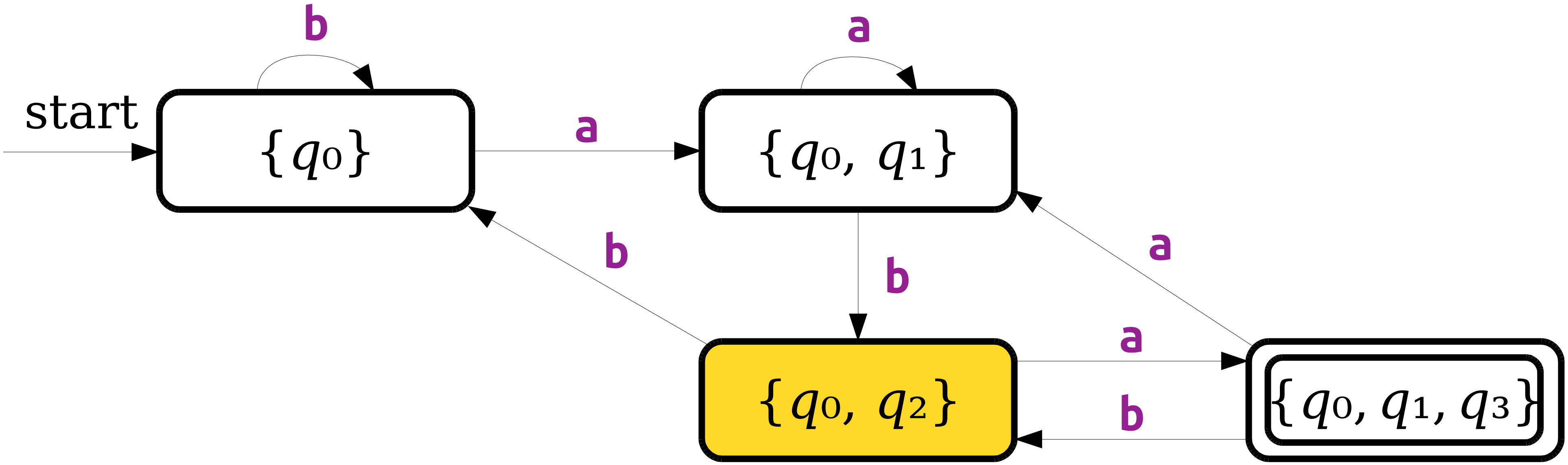


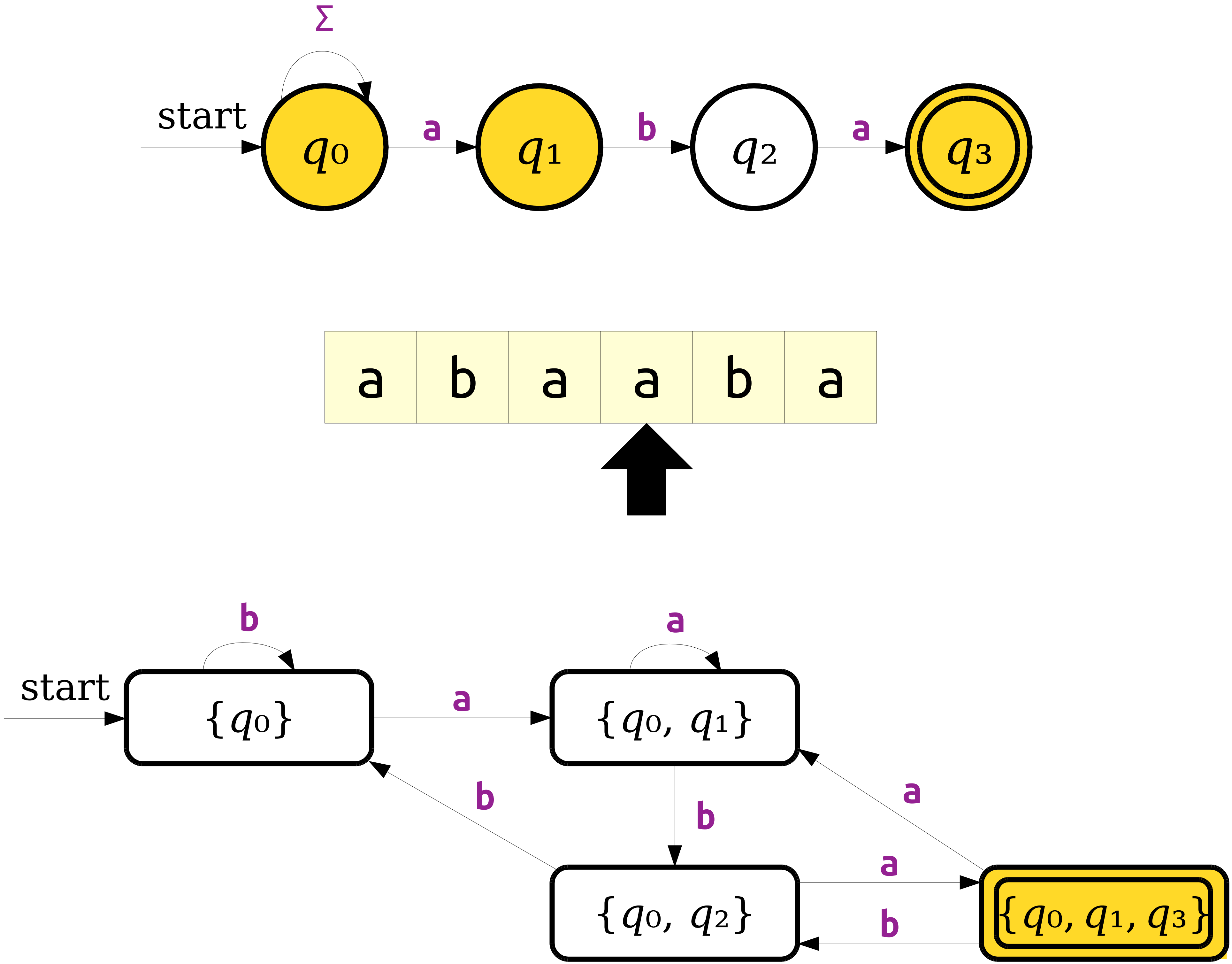
a	b	a	a	b	a



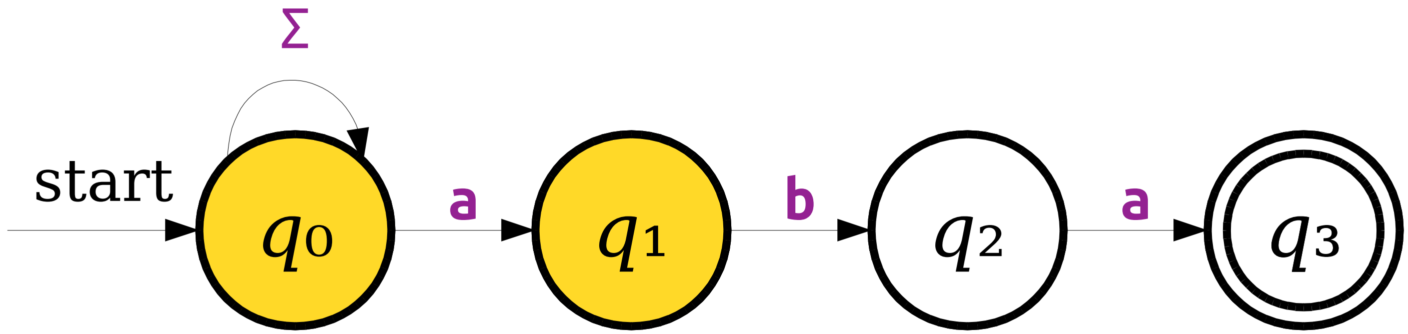

Σ
start
q_0
a
q_1
b
q_2
a
q_3


a
b
a
a
b
a

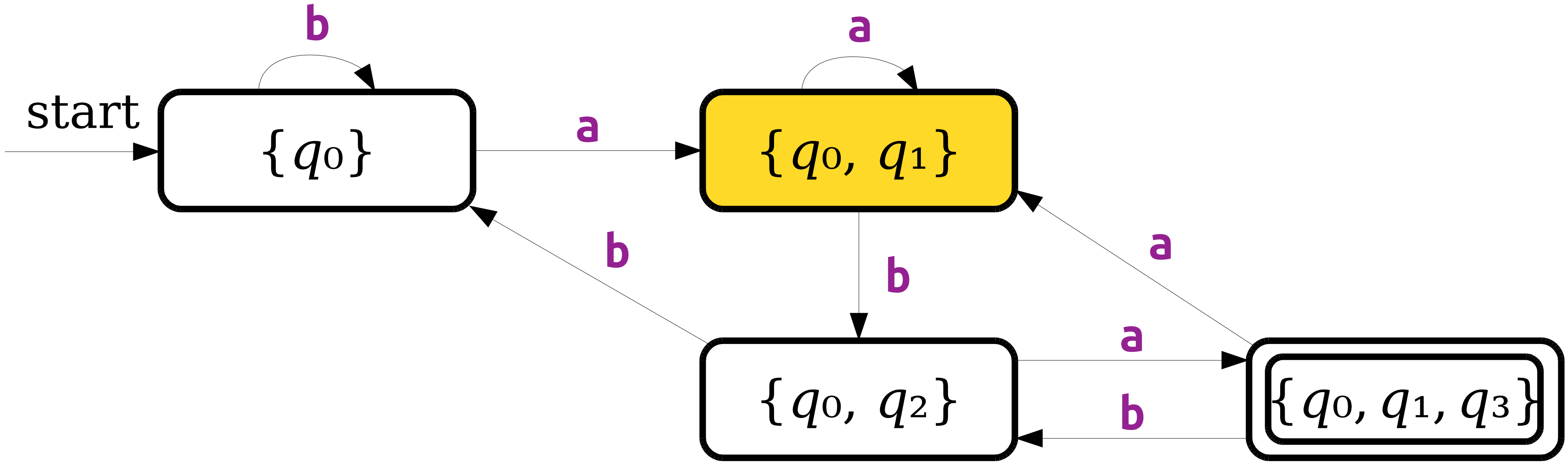

b
start
$\{q_0\}$
a
a
$\{q_0, q_1\}$
b
b
a
$\{q_0, q_2\}$
a
b
$\{q_0, q_1, q_3\}$

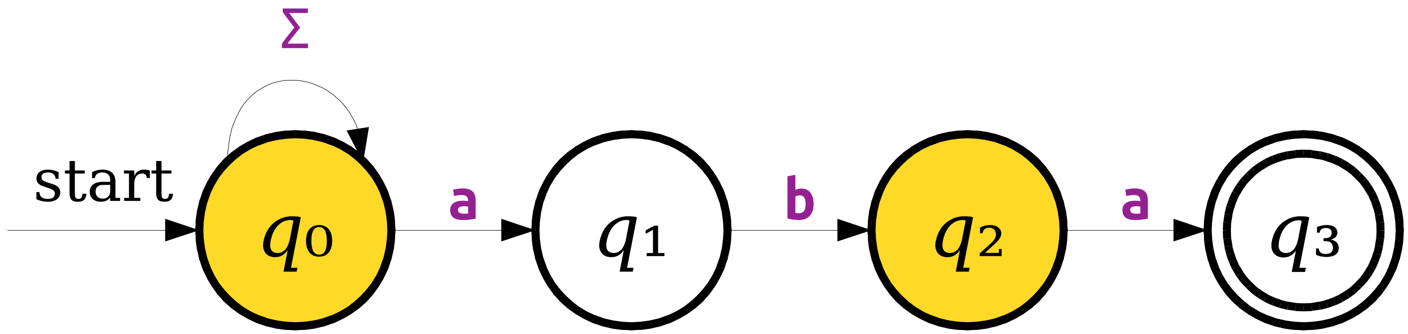


a	b	a	a	b	a

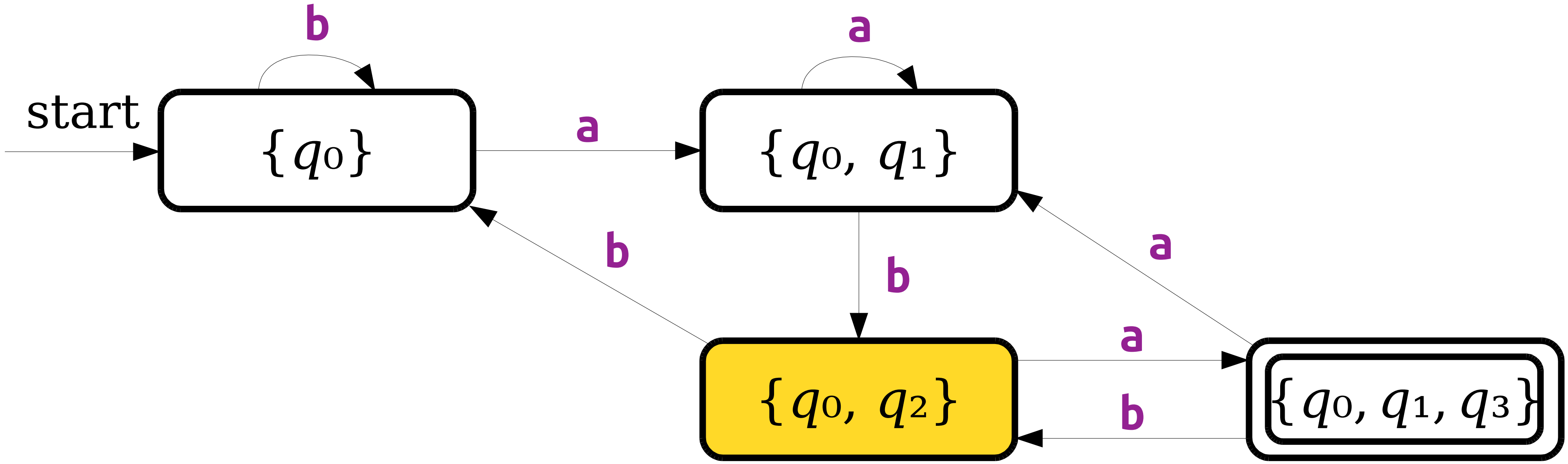


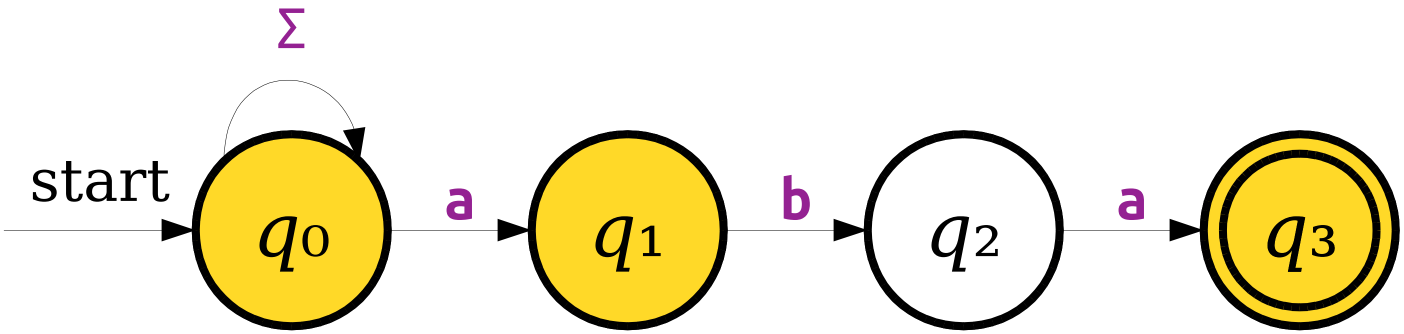
a	b	a	a	b	a



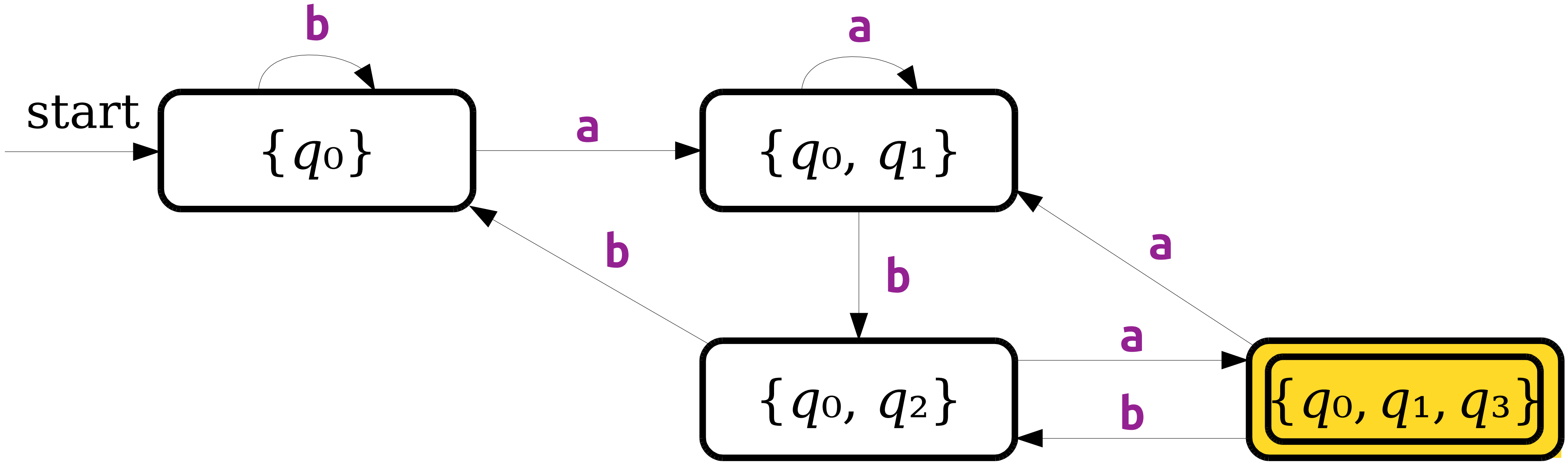


a	b	a	a	b	a





a	b	a	a	b	a



The Subset Construction

- This procedure for turning an NFA for a language L into a DFA for a language L is called the ***subset construction***.
  - It's sometimes called the ***powerset construction***; it's different names for the same thing!
- Intuitively:
  - Each state in the DFA corresponds to a set of states from the NFA.
  - Each transition in the DFA corresponds to what transitions would be taken in the NFA when using the massive parallel intuition.
  - The accepting states in the DFA correspond to which sets of states would be considered accepting in the NFA when using the massive parallel intuition.
- There's an online ***Guide to the Subset Construction*** with a more elaborate example involving ε-transitions and cases where the NFA dies; check that for more details.

The Subset Construction

- In converting an NFA to a DFA, the DFA's states correspond to sets of NFA states.
- ***Useful fact:*** $|\wp(S)| = 2^{|S|}$ for any finite set S.
- In the worst-case, the construction can result in a DFA that is *exponentially larger* than the original NFA.
- ***Question to ponder:*** Can you find a family of languages that have NFAs of size n, but no DFAs of size less than 2^n?

The Regular Languages

Regular Languages

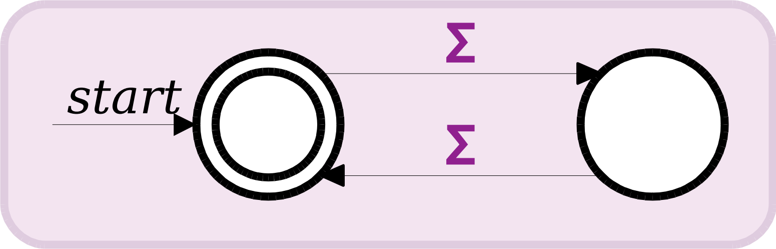
- Let $L \subseteq \Sigma^*$ be a language.
- We say that L is a ***regular language*** if there is a DFA D where $\mathscr{L}(D) = L$.
- Equivalently, L is a regular language if there is an NFA N where $\mathscr{L}(N) = L$.
- Key questions:
    - What do the regular languages “feel” like?
    - What properties do they have?
    - What languages *aren’t* regular?

Closure Under Union

- If L_1 and L_2 are languages over the alphabet Σ, the language $L_1 \cup L_2$ is the language of all strings in at least one of the two languages.
- If L_1 and L_2 are regular languages, is $L_1 \cup L_2$?

$L_1 = \{ w \in \{\texttt{a}, \texttt{b}\}^* \mid w$ has even length $\}$
$L_2 = \{ w \in \{\texttt{a}, \texttt{b}\}^* \mid w$ has length exactly three $\}$

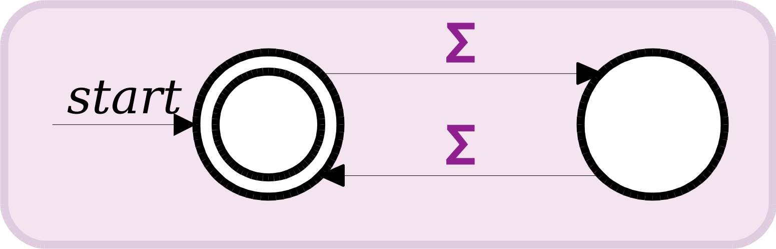
Construct an NFA for $L_1 \cup L_2$.



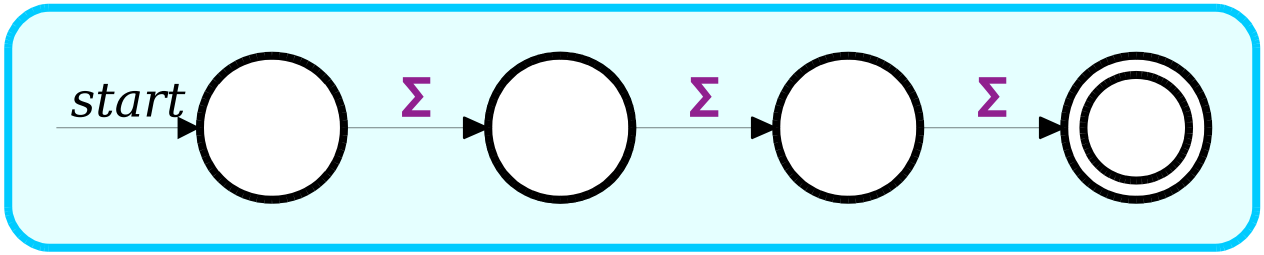
DFA for L_1

$L_1 = \{ w \in \{\mathtt{a}, \mathtt{b}\}^* \mid w \text{ has even length } \}$
$L_2 = \{ w \in \{\mathtt{a}, \mathtt{b}\}^* \mid w \text{ has length exactly three } \}$

Construct an NFA for $L_1 \cup L_2$.



DFA for L_1

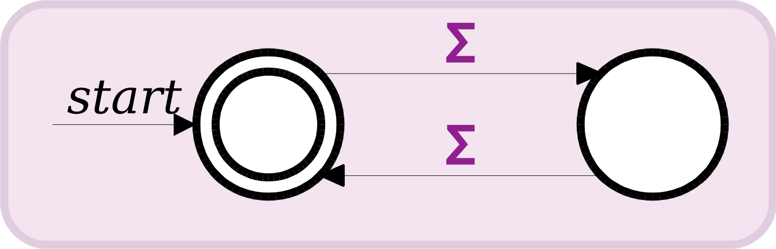


NFA for L_2

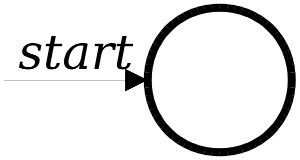
$L_1 = \{\ w \in \{\texttt{a}, \texttt{b}\}^* \mid w \text{ has even length }\}$

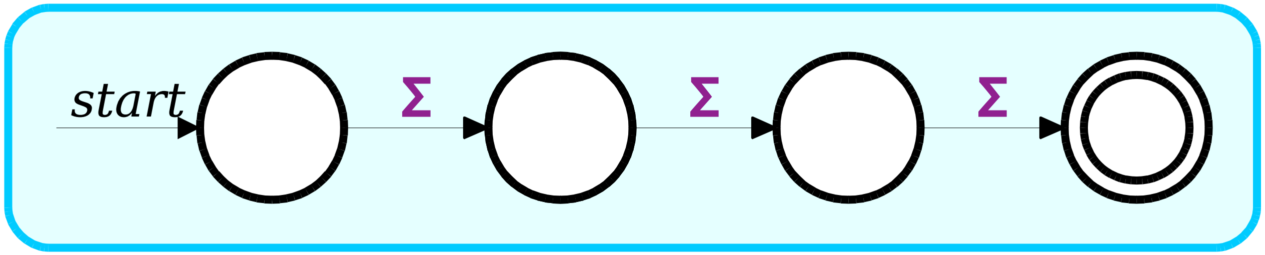
$L_2 = \{\ w \in \{\texttt{a}, \texttt{b}\}^* \mid w \text{ has length exactly three }\}$

Construct an NFA for $L_1 \cup L_2$.



DFA for L_1

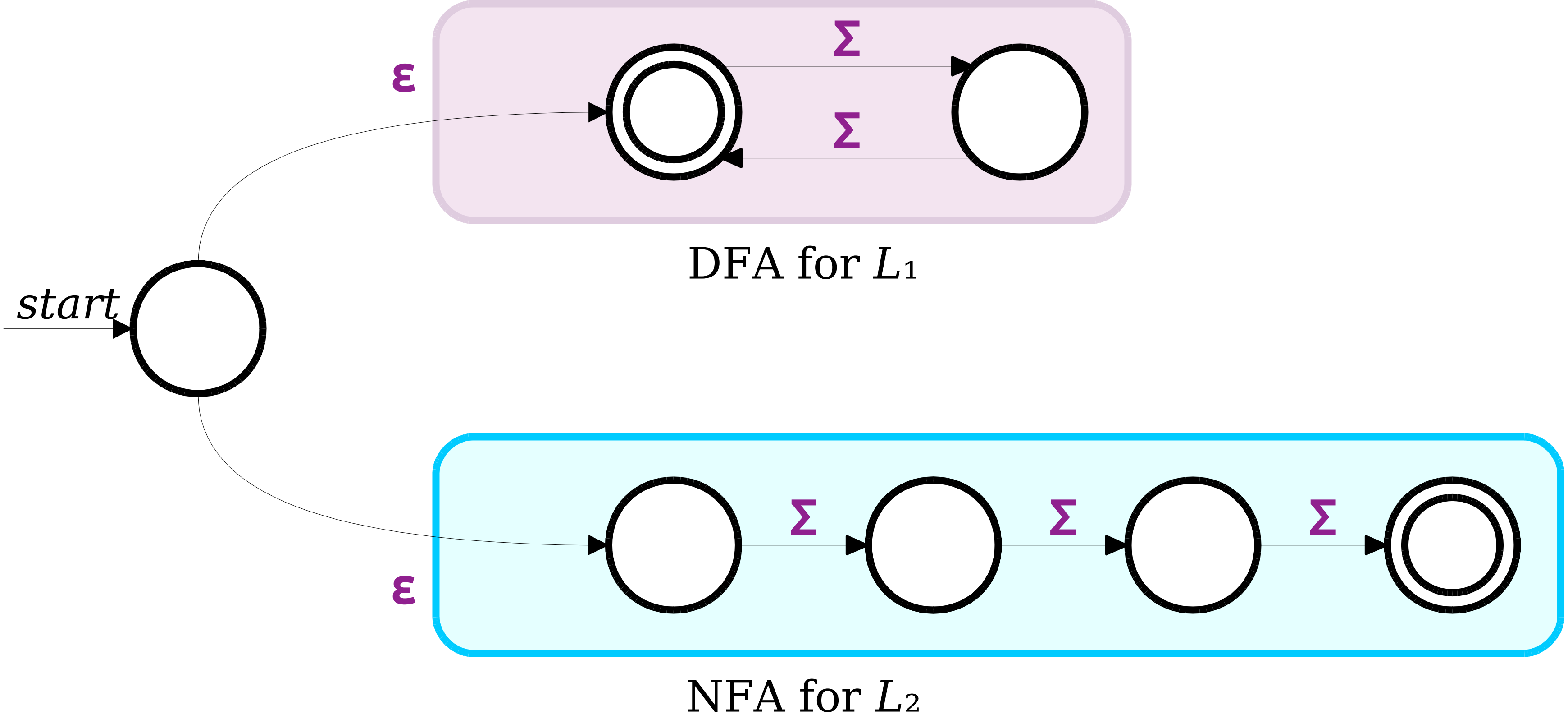




NFA for L_2

$$L_1 = \{ \ w \in \{\texttt{a}, \texttt{b}\}^* \mid w \text{ has even length } \}$$
$$L_2 = \{ \ w \in \{\texttt{a}, \texttt{b}\}^* \mid w \text{ has length exactly three } \}$$

Construct an NFA for $L_1 \cup L_2$.



$L_1 = \{ w \in \{\texttt{a}, \texttt{b}\}^* \mid w \text{ has even length } \}$
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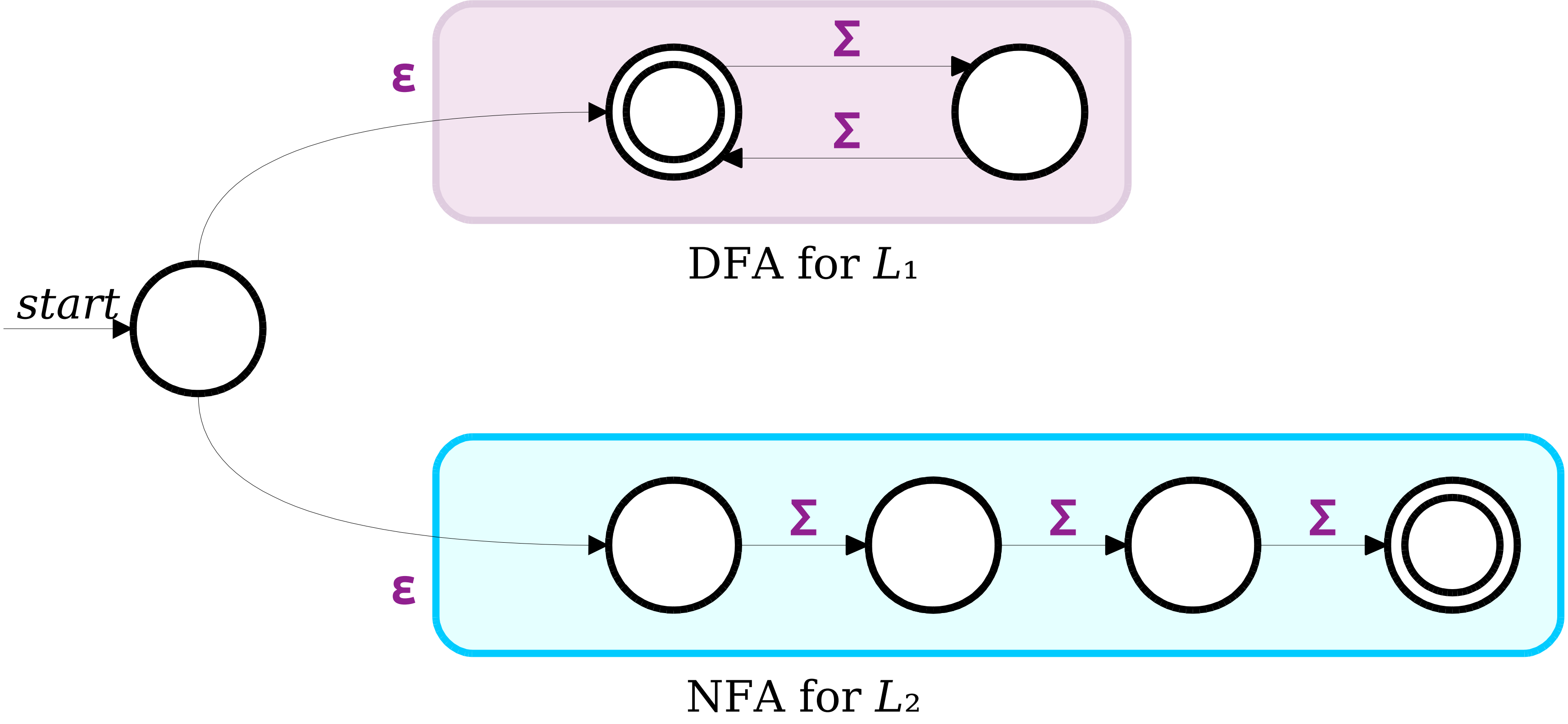


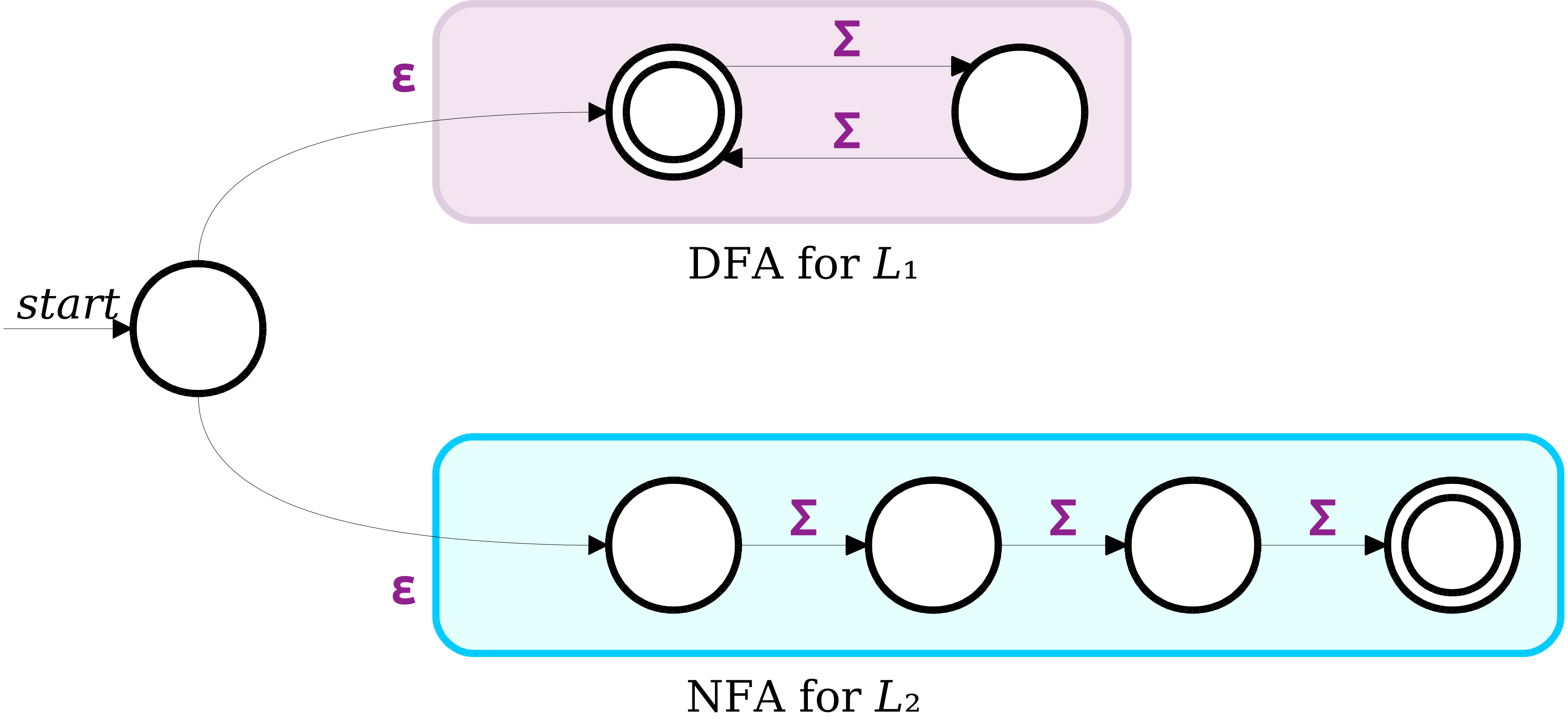
DFA for L_1

NFA for L_2

a	b	a	b	a	b

L_1 = { w ∈ {a, b}* | w has even length }
L_2 = { w ∈ {a, b}* | w has length exactly three }

Construct an NFA for $L_1 \cup L_2$.



a	b	a	b	a	b

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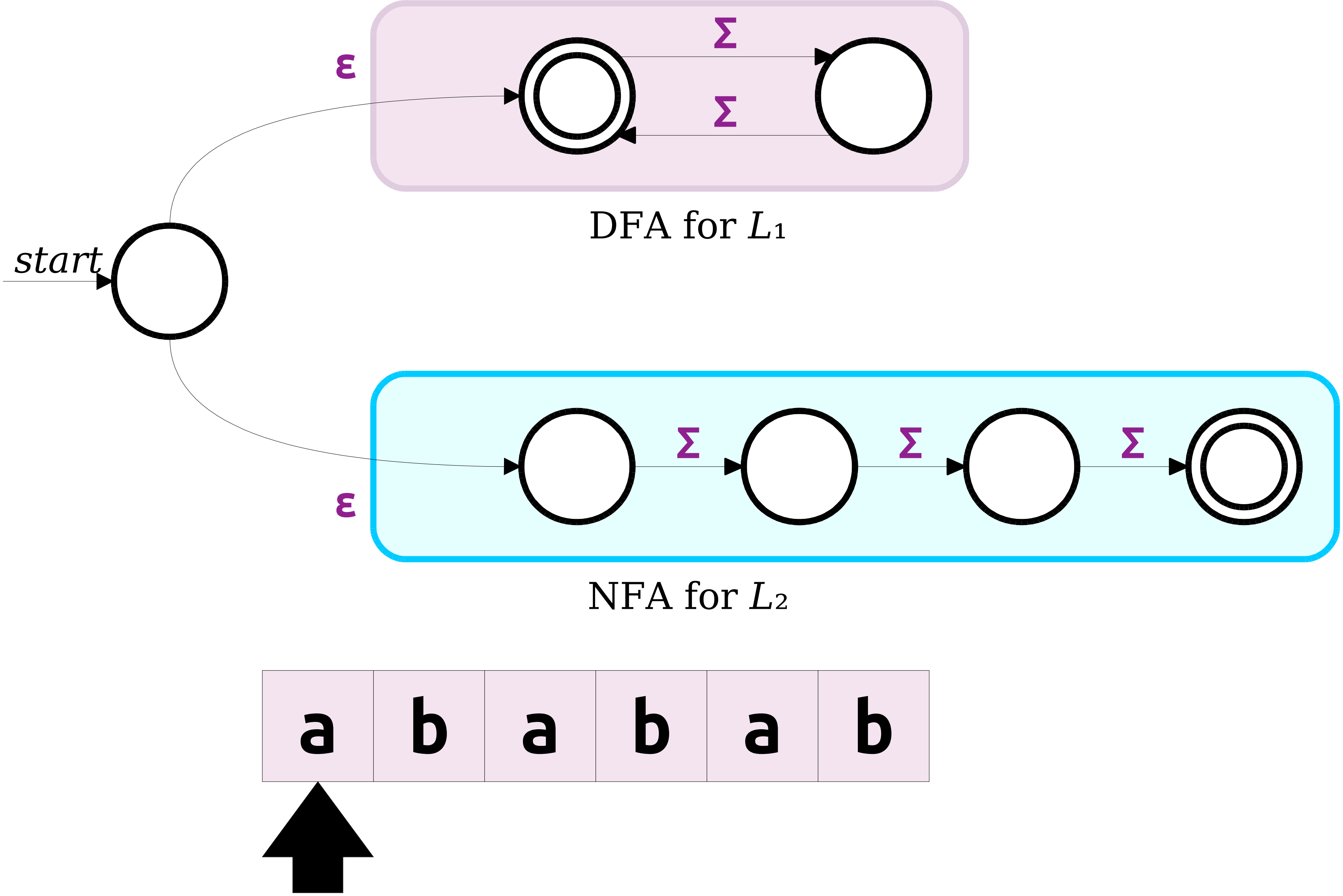

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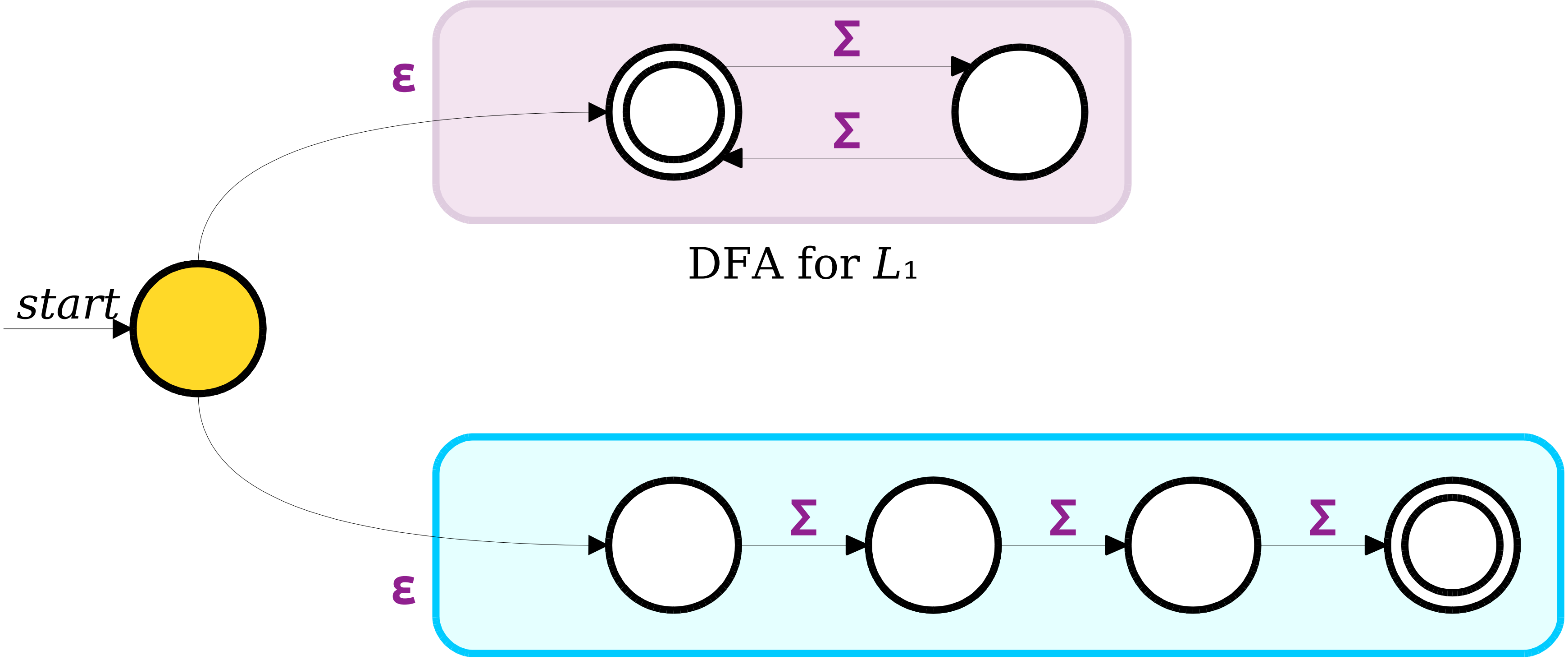
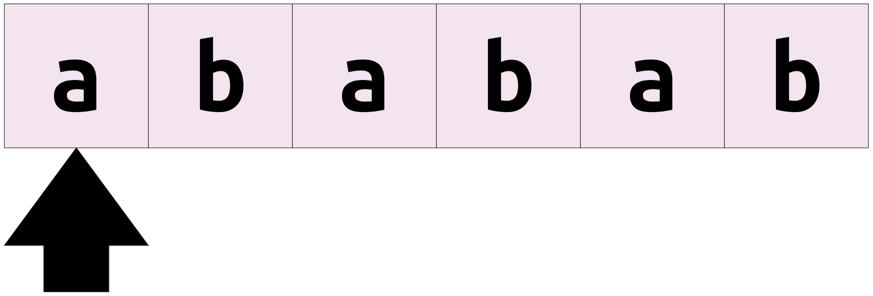


DFA for L_1

NFA for L_2



L_1 = { w ∈ {**a**, **b**}* | w has even length }
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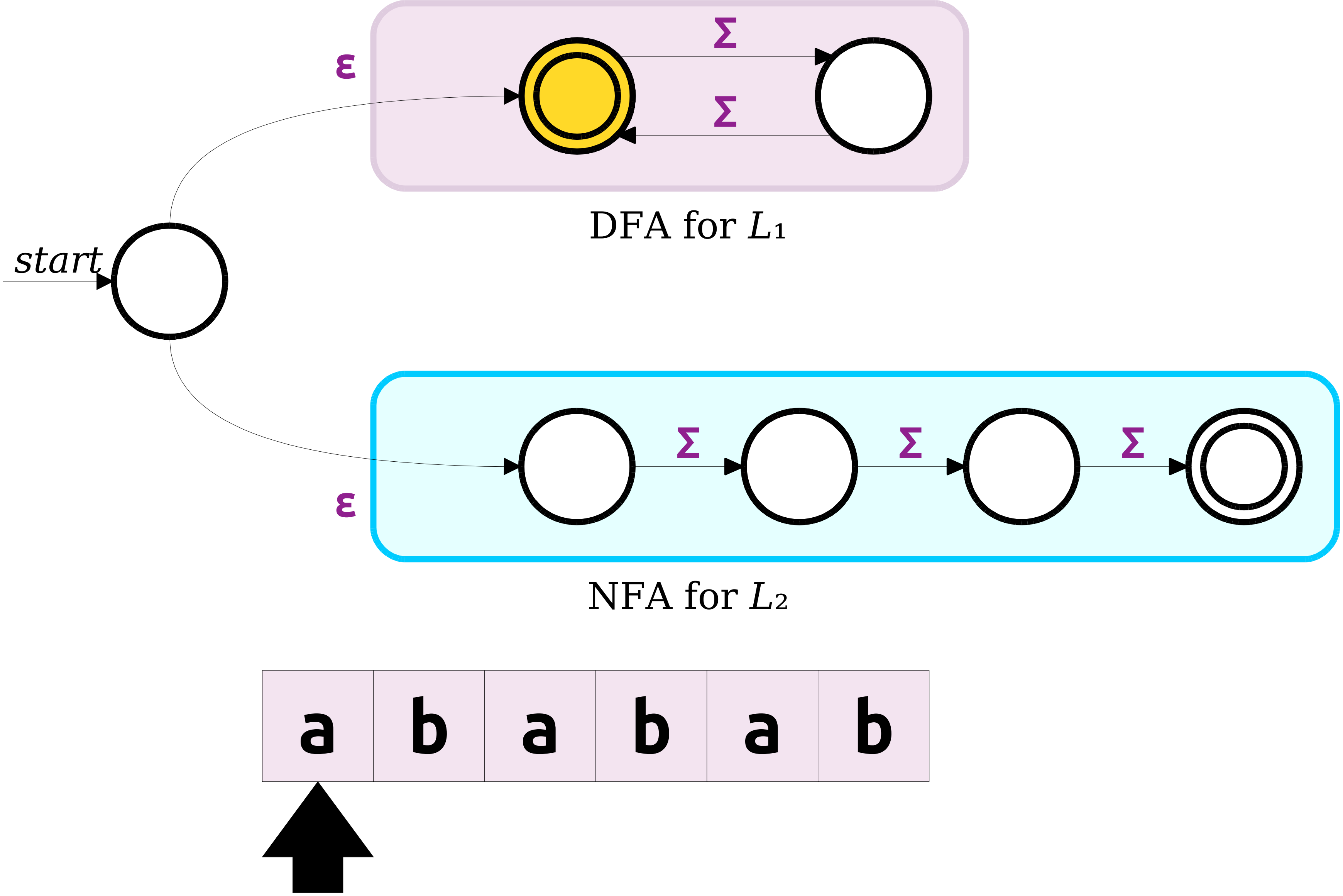
Construct an NFA for $L_1 \cup L_2$.



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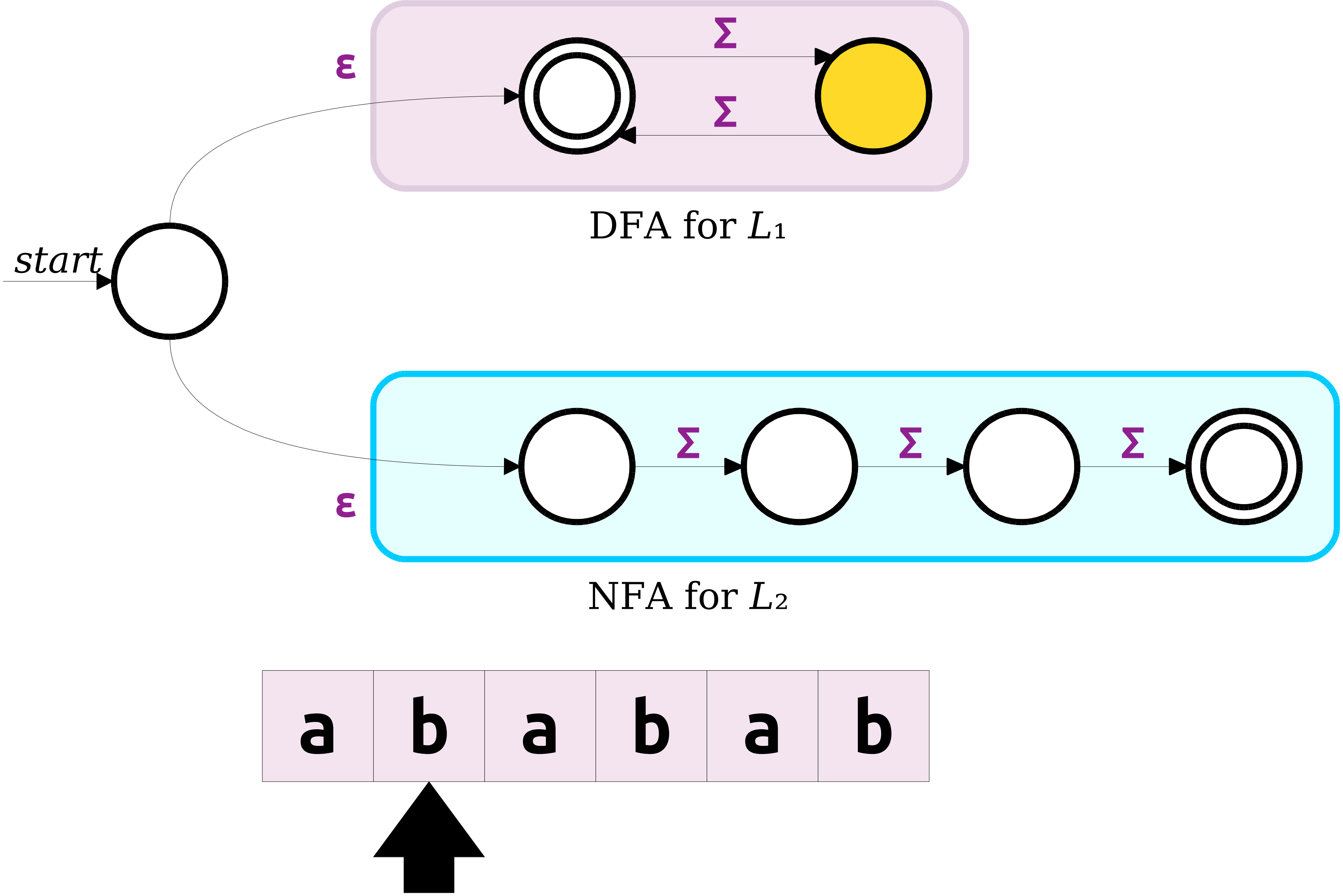

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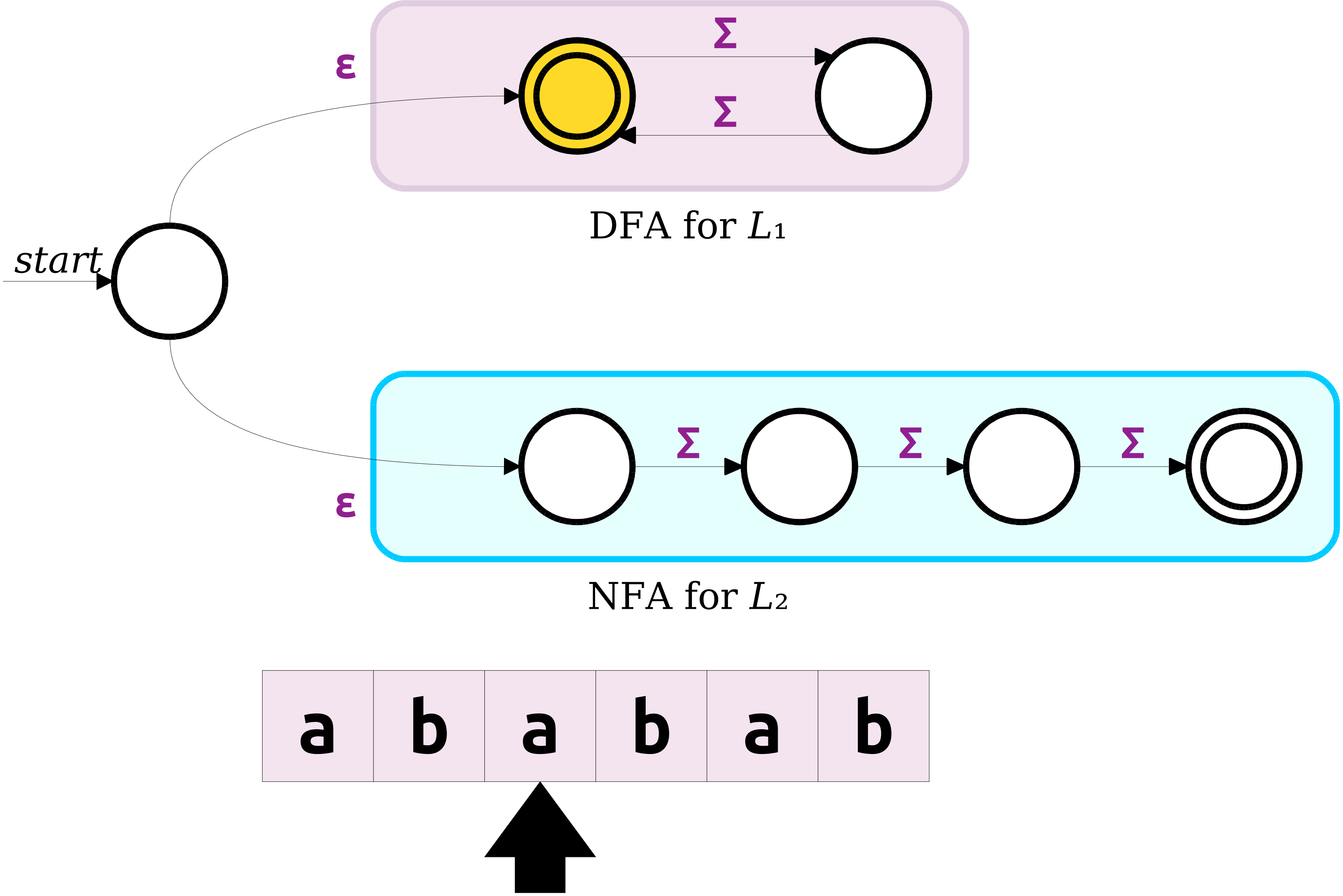

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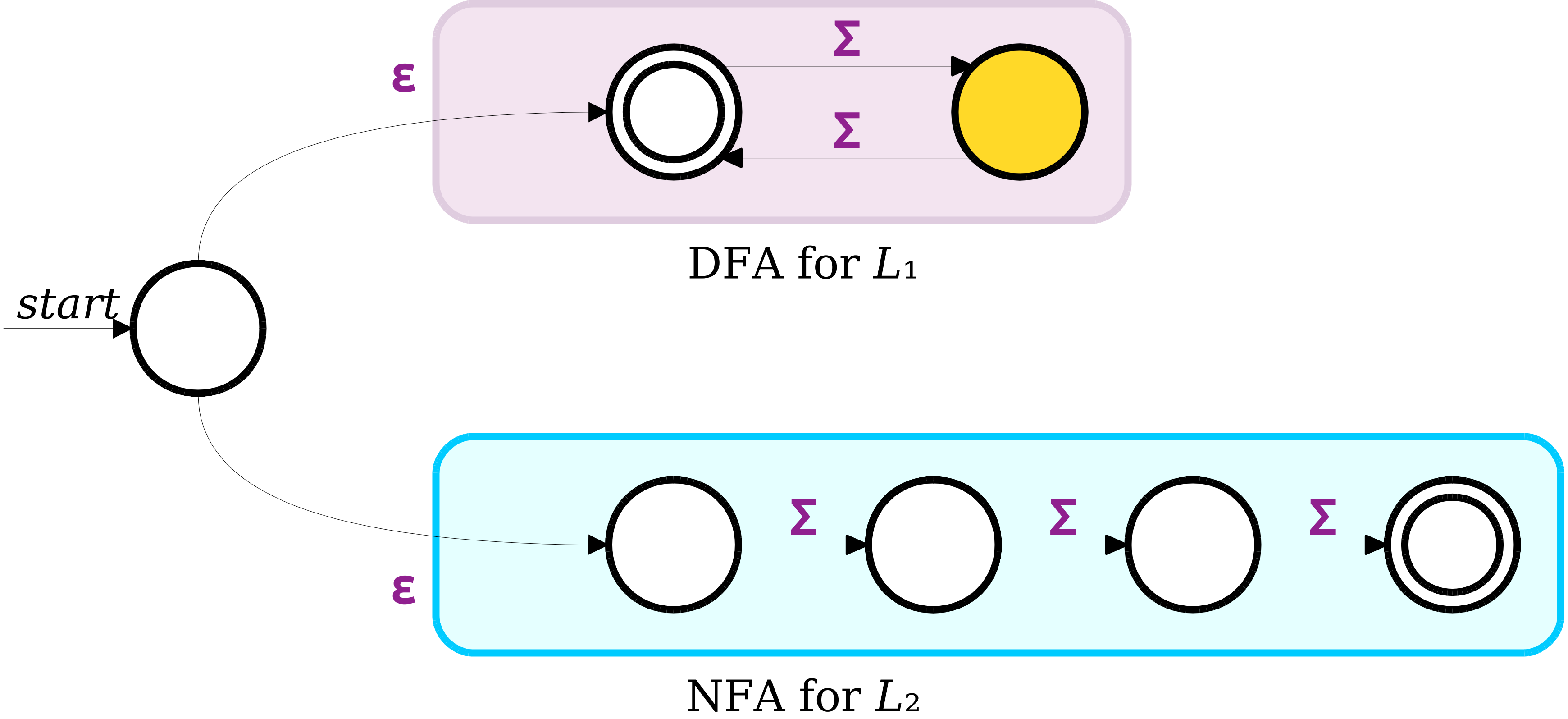

a	b	a	b	a	b

L_1 = { w ∈ {a, b}* | w has even length }
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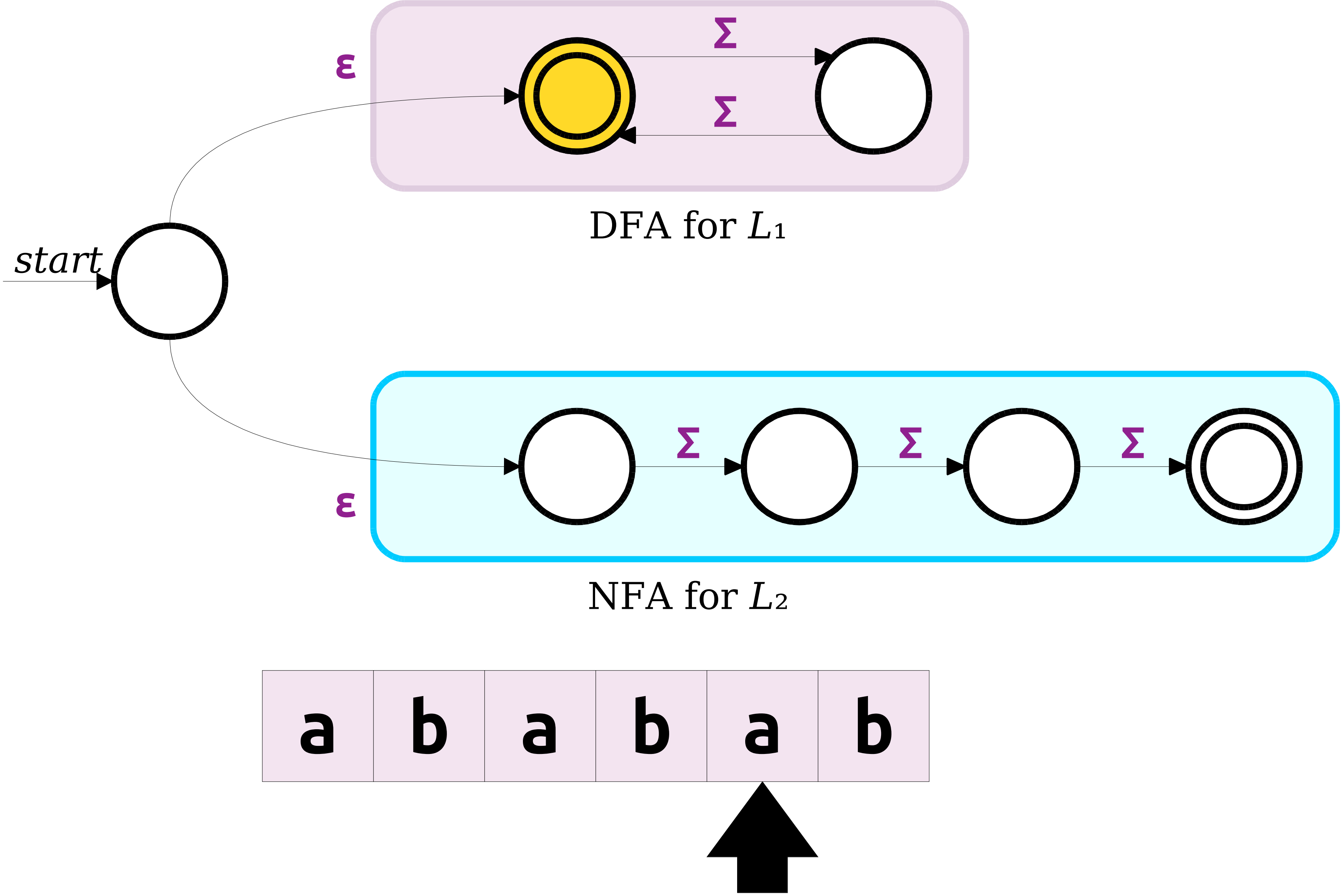
Construct an NFA for $L_1 \cup L_2$.



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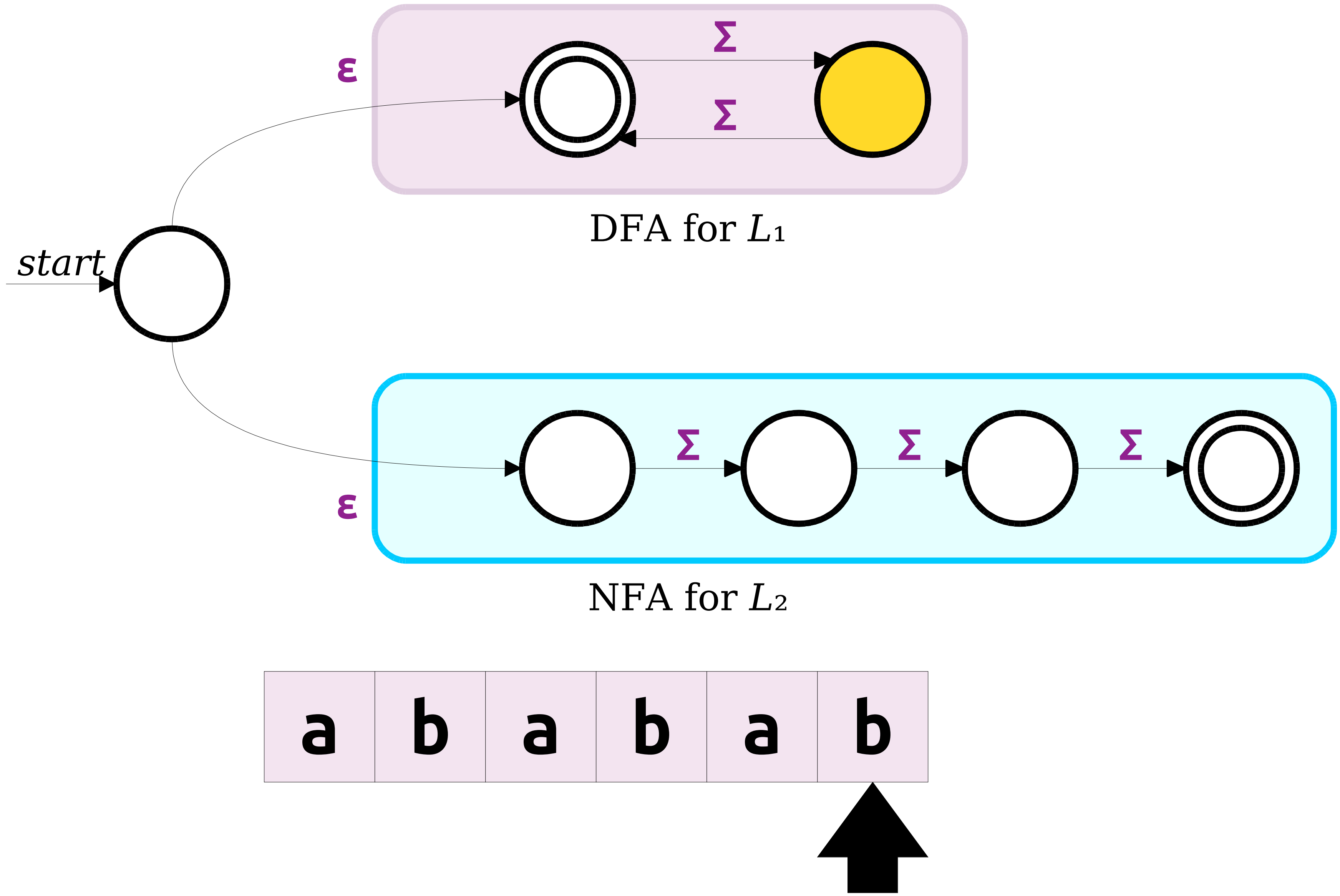

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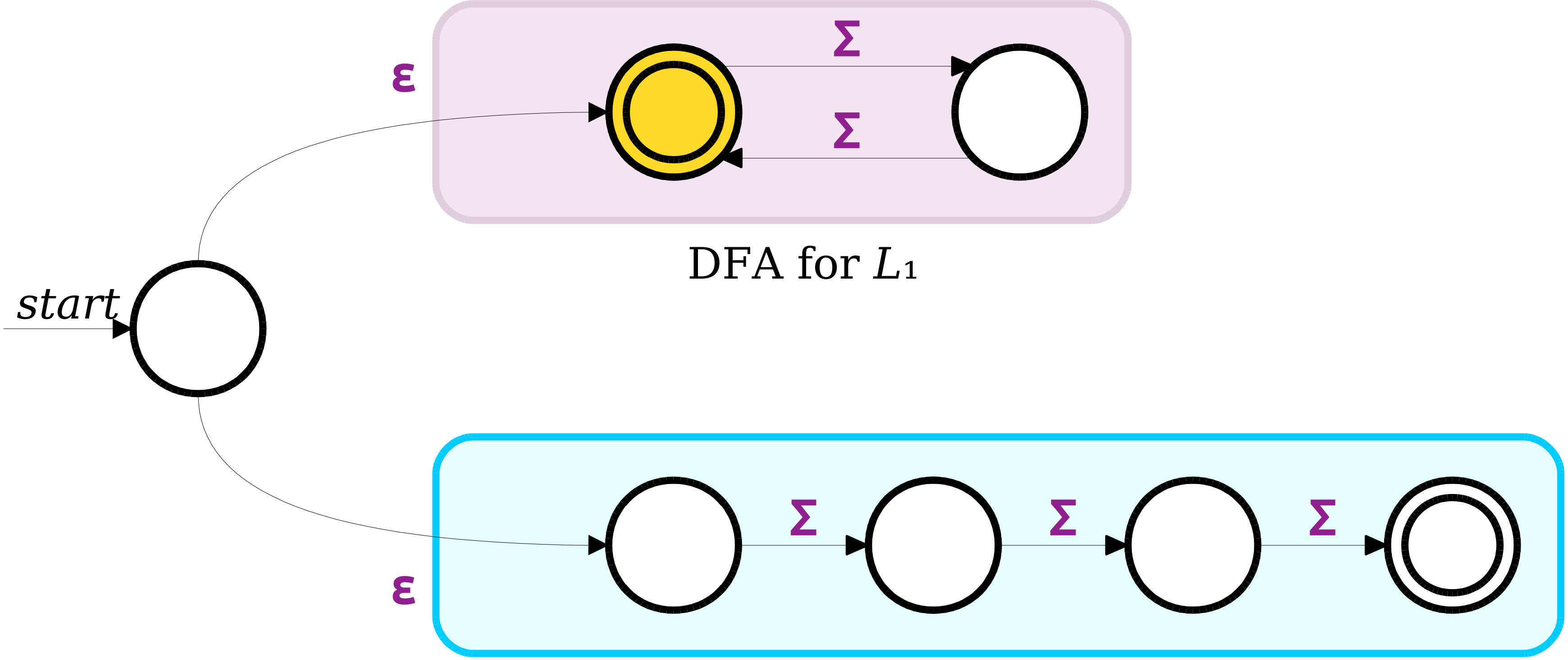

DFA for L_1

NFA for L_2

a	b	a	b	a	b

L_1 = { w ∈ {a, b}* | w has even length }
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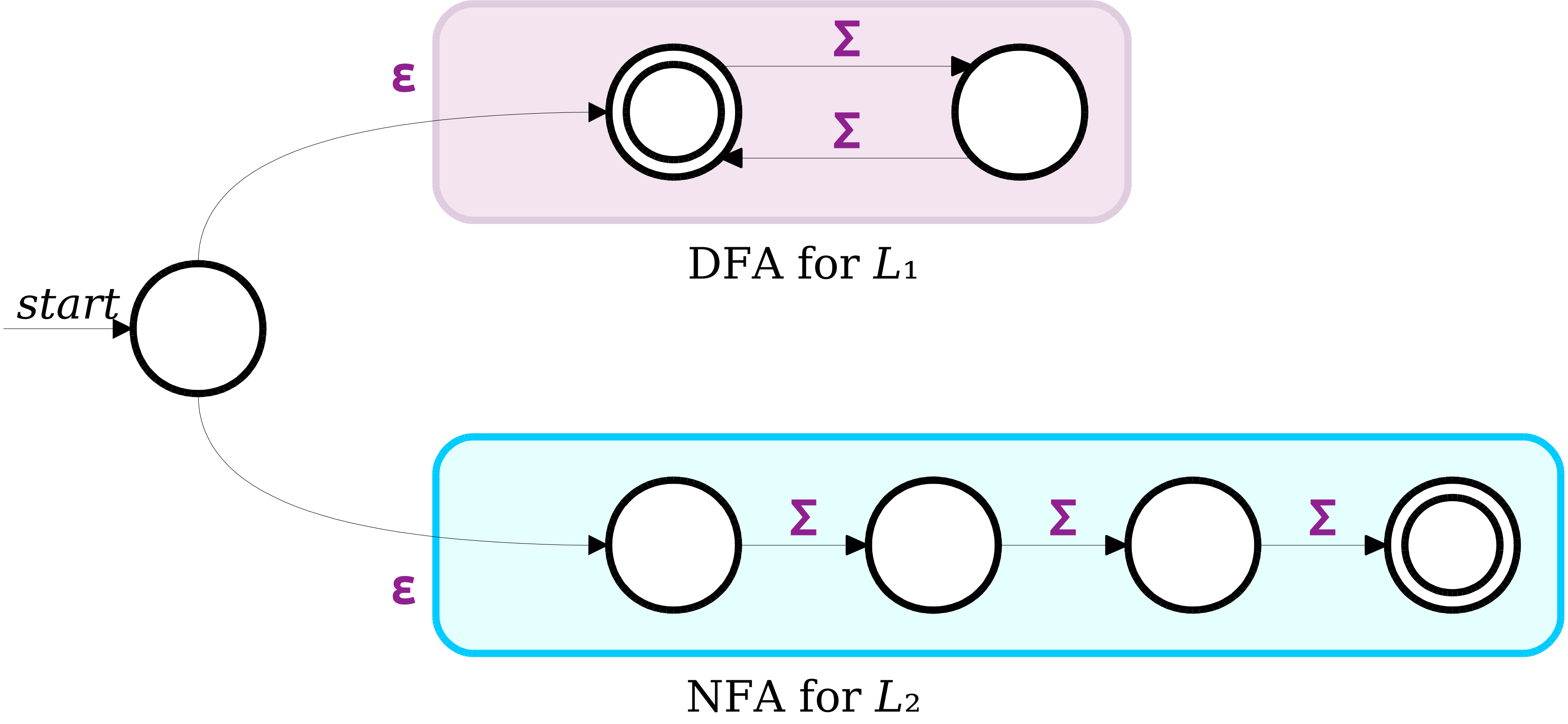
Construct an NFA for $L_1 \cup L_2$.



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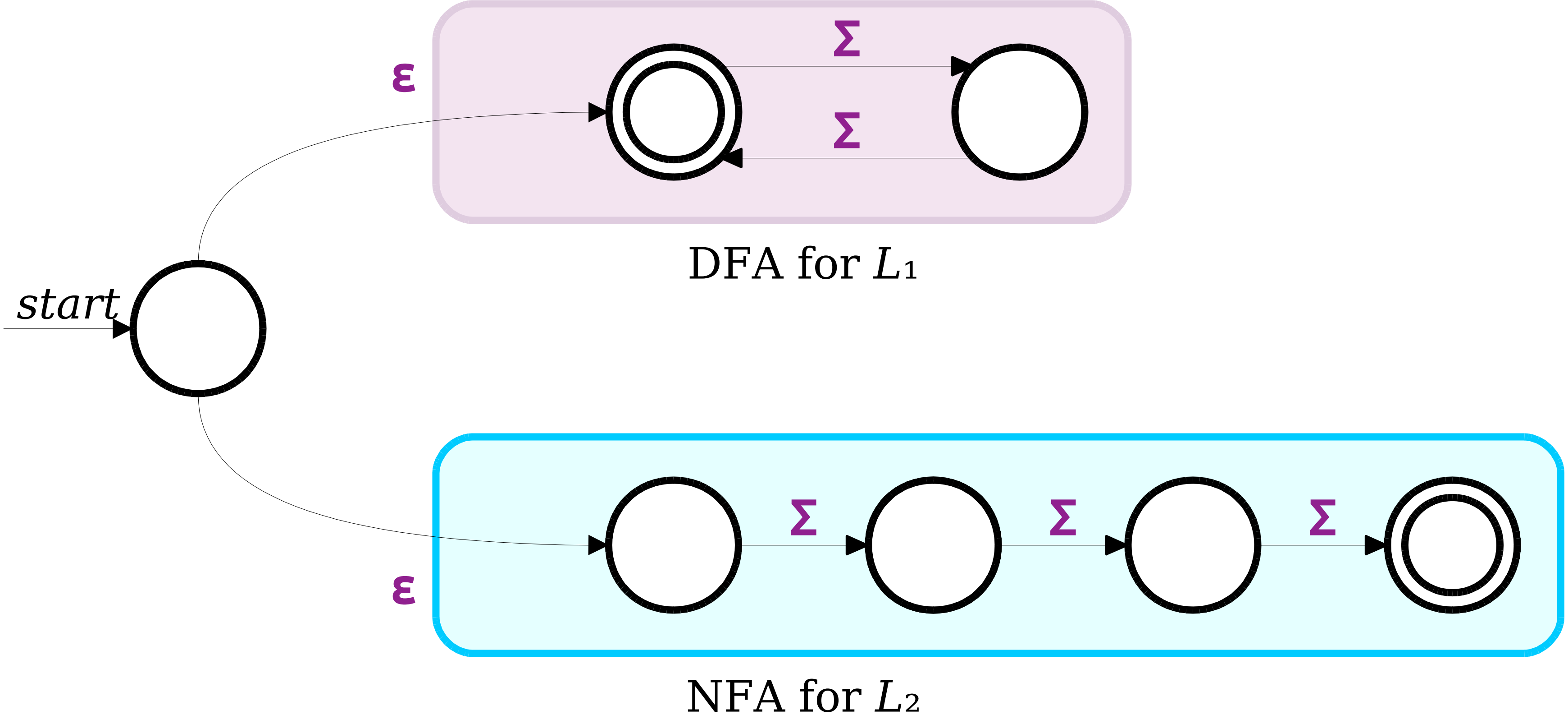

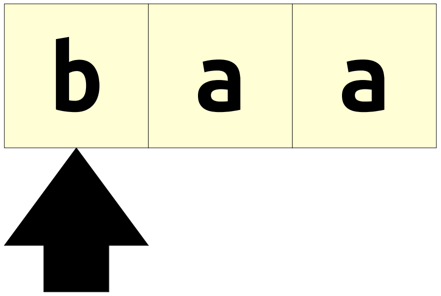

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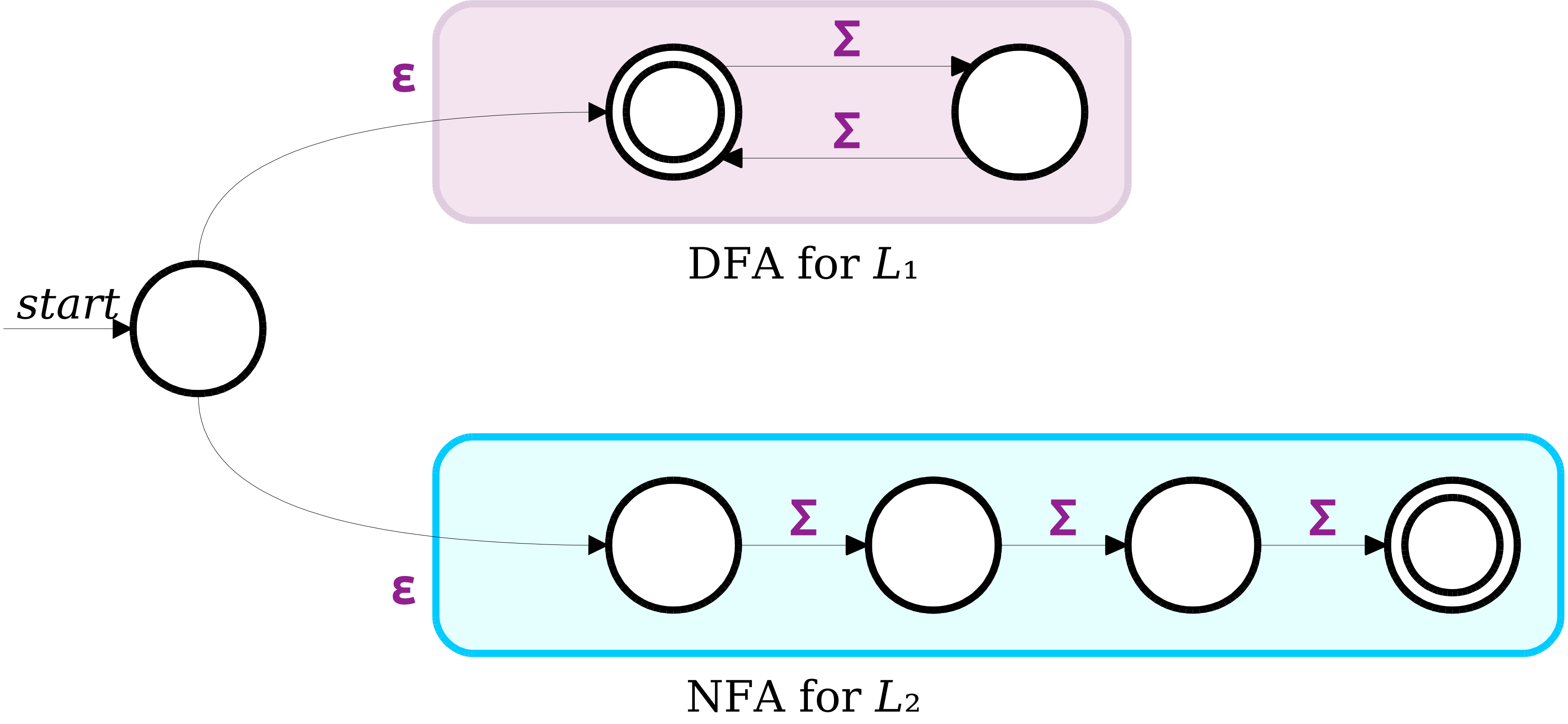

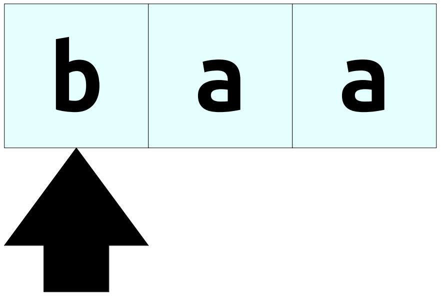

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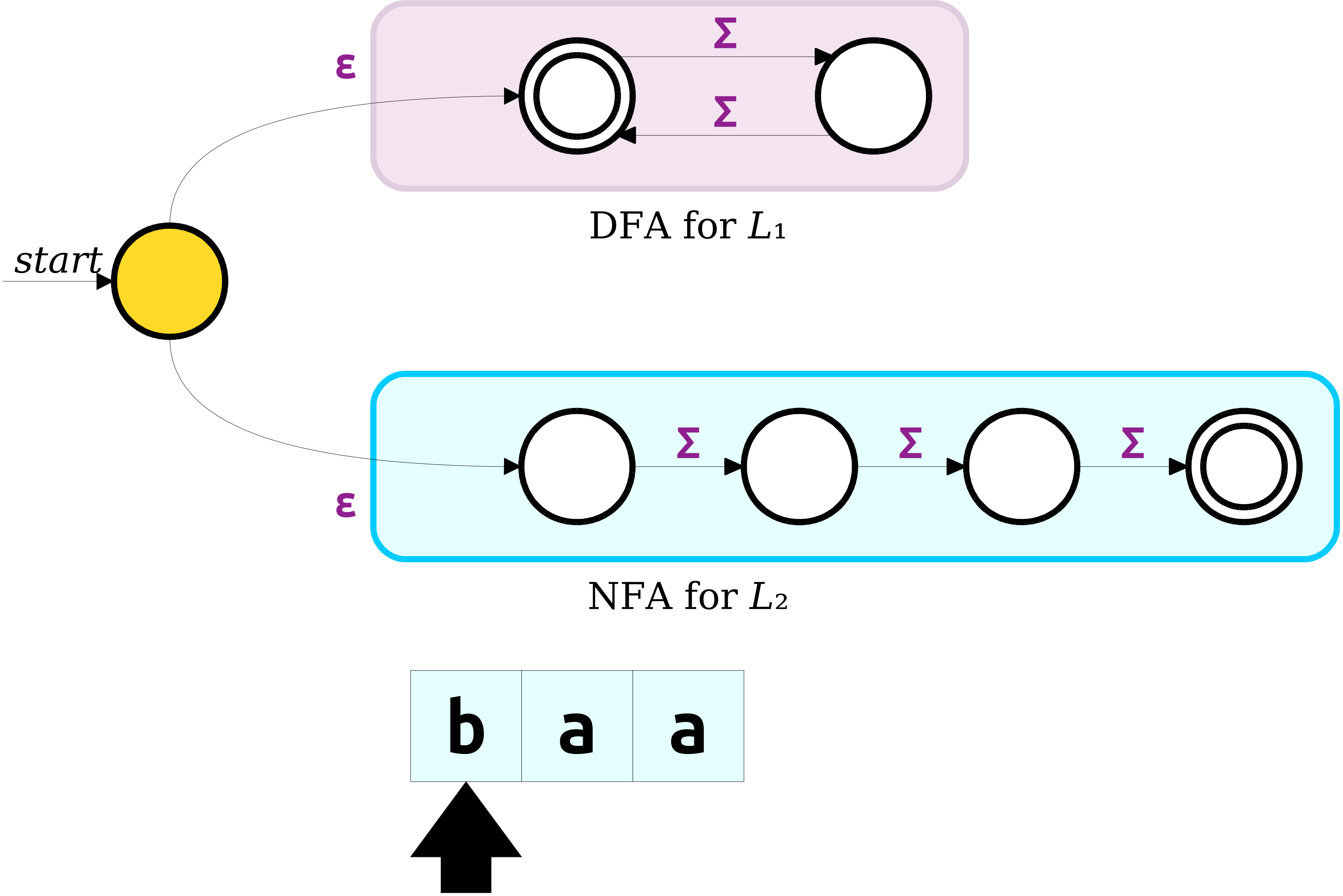

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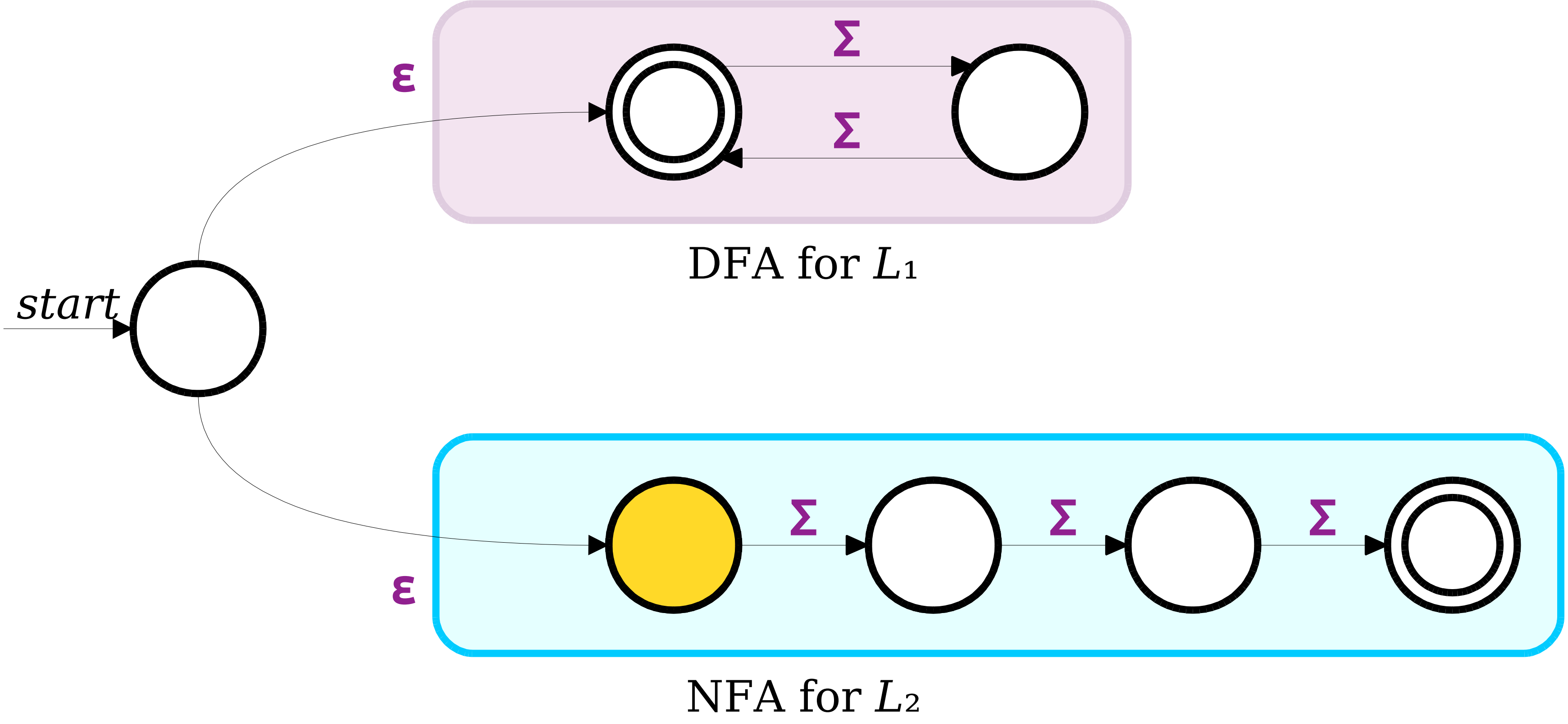
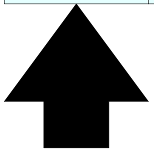


b	a	a



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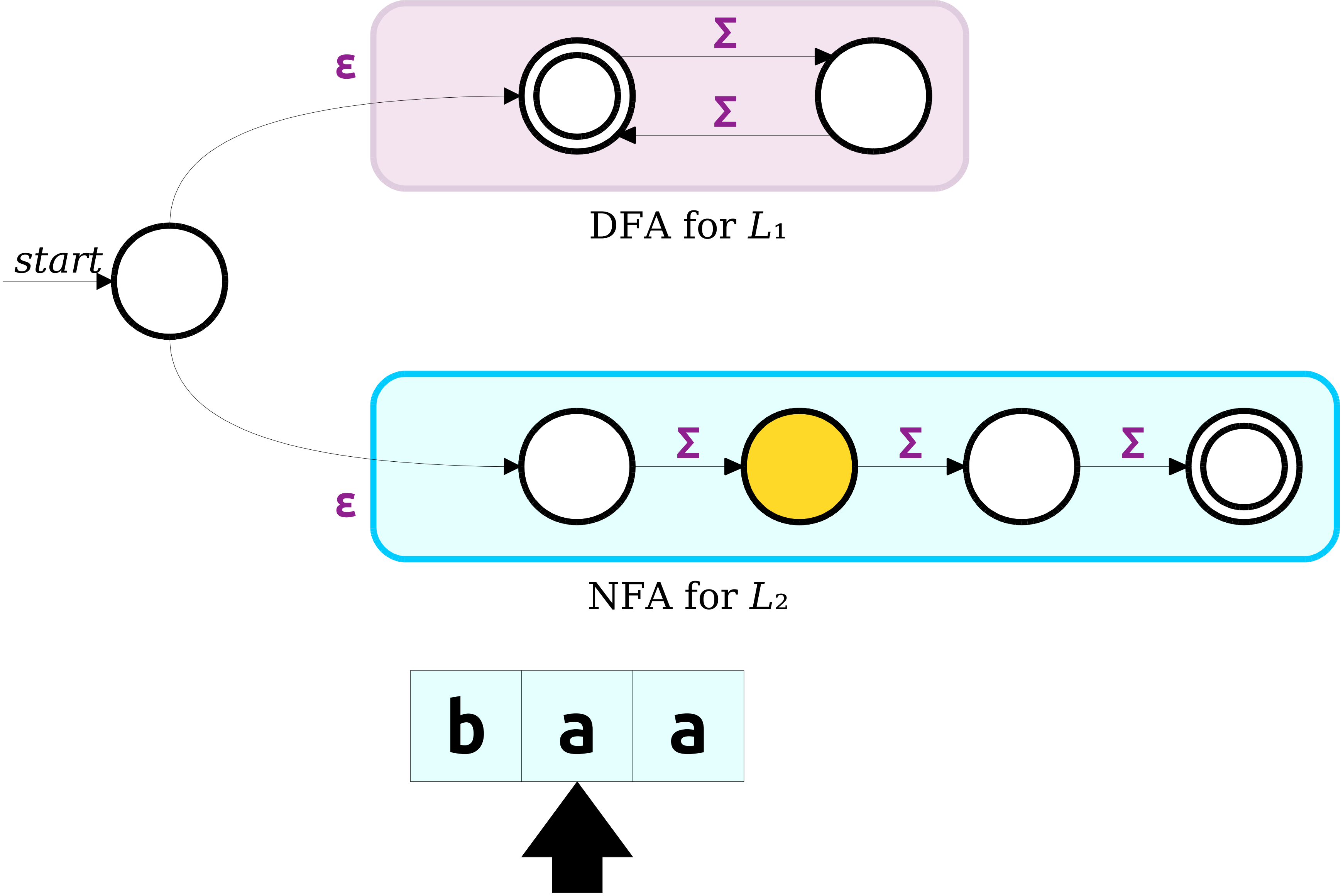

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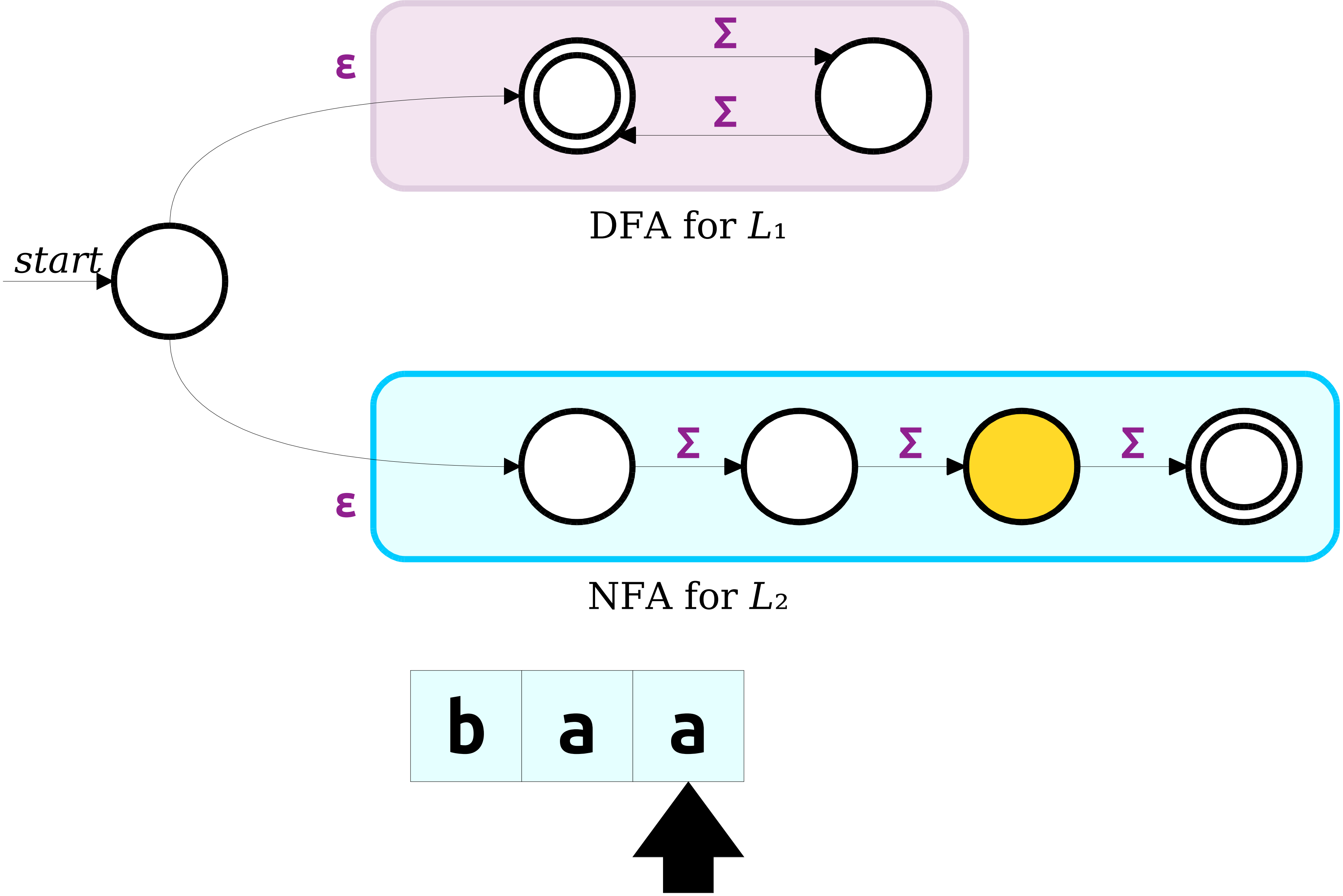
L_2 = { w ∈ {a, b}* | w has length exactly three }

Construct an NFA for $L_1 \cup L_2$.



L_1 = { w ∈ {a, b}* | w has even length }
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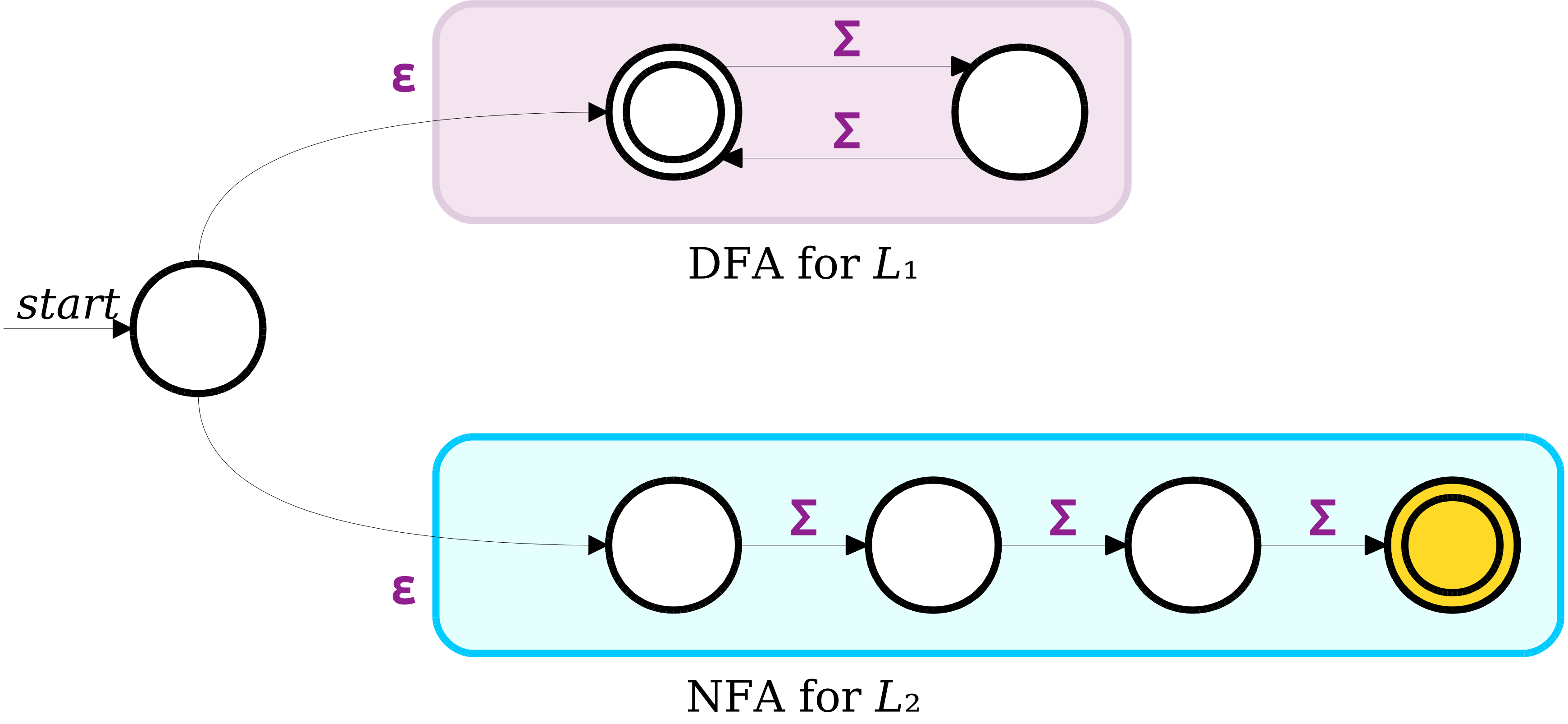

b	a	a

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Closure Under Intersection

- If L_1 and L_2 are languages over Σ, then $L_1 \cap L_2$ is the language of strings in both L_1 and L_2.
- Question: If L_1 and L_2 are regular, is $L_1 \cap L_2$ regular as well?

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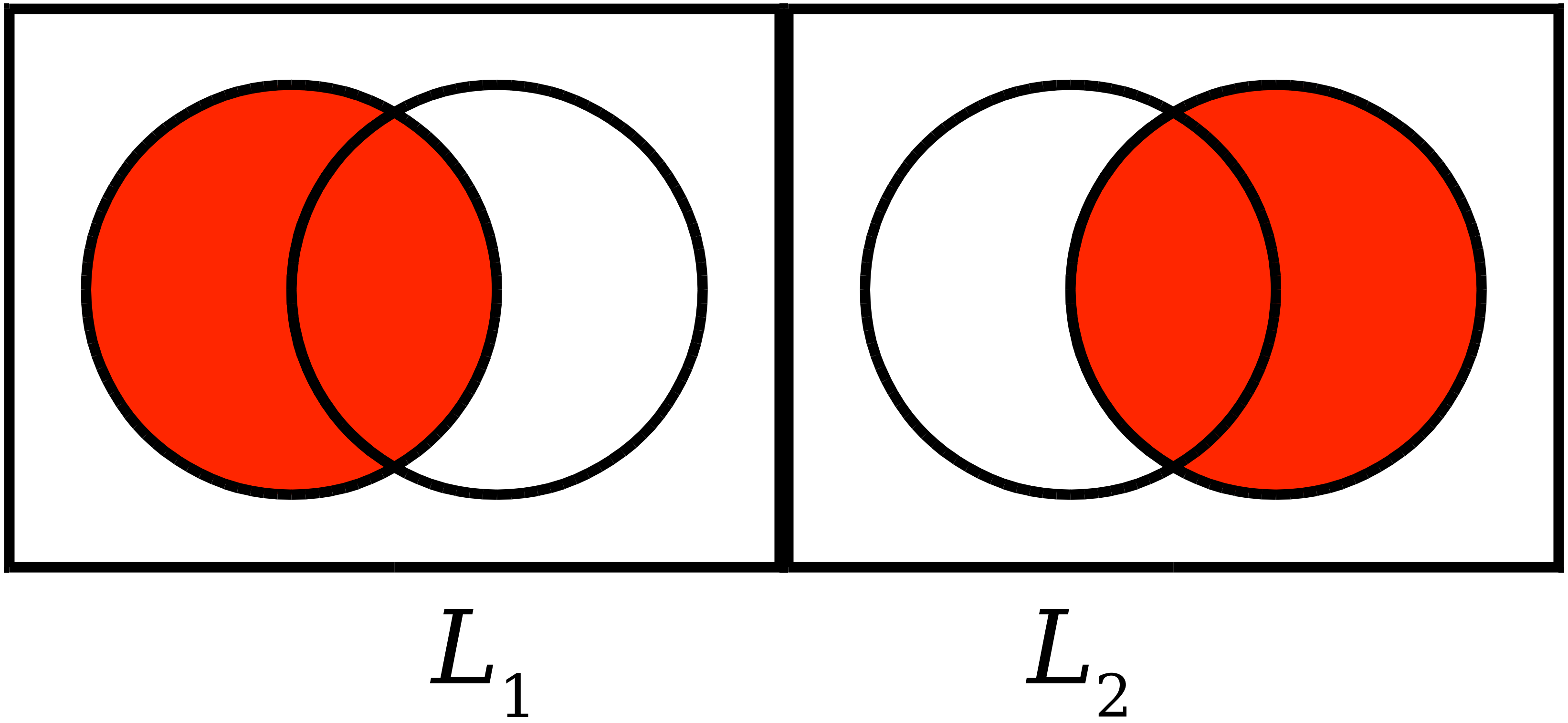

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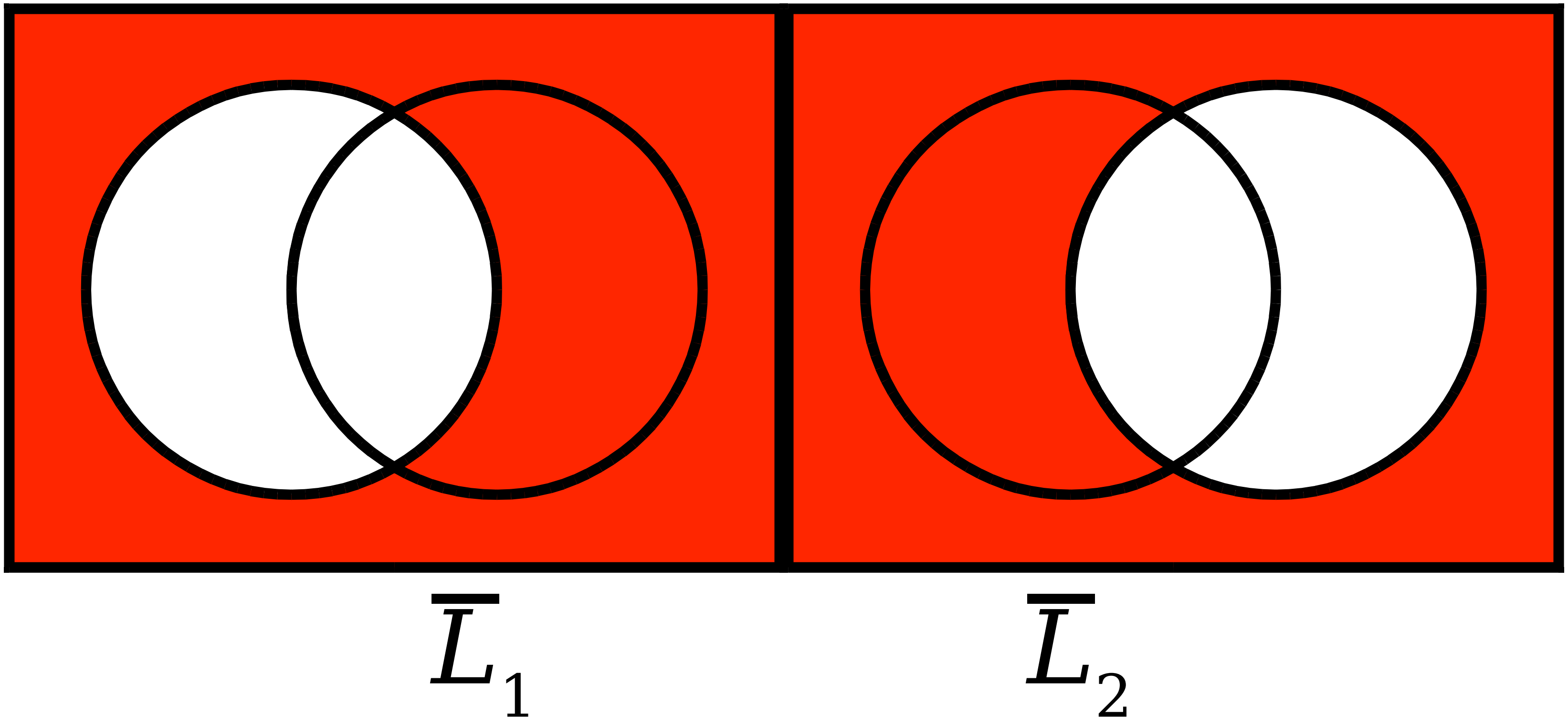

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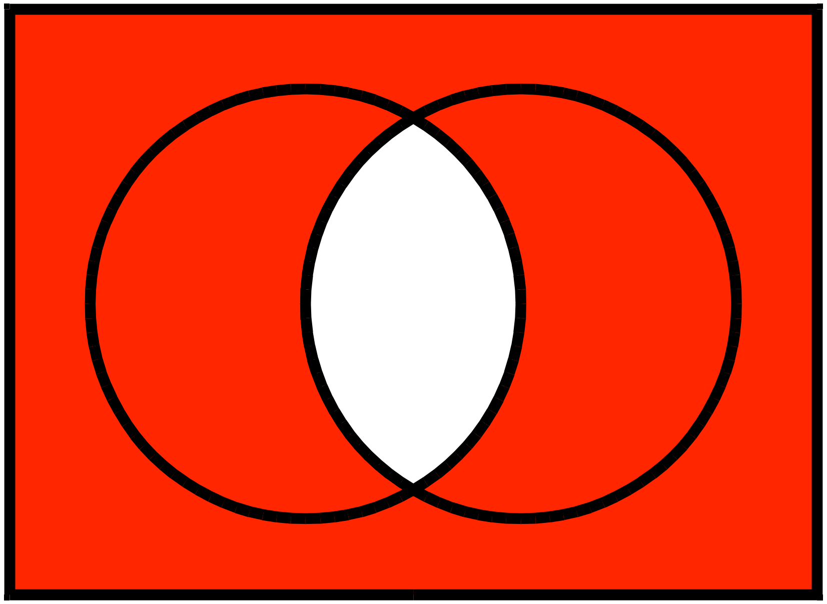

$$\overline{L}_1 \cup \overline{L}_2$$

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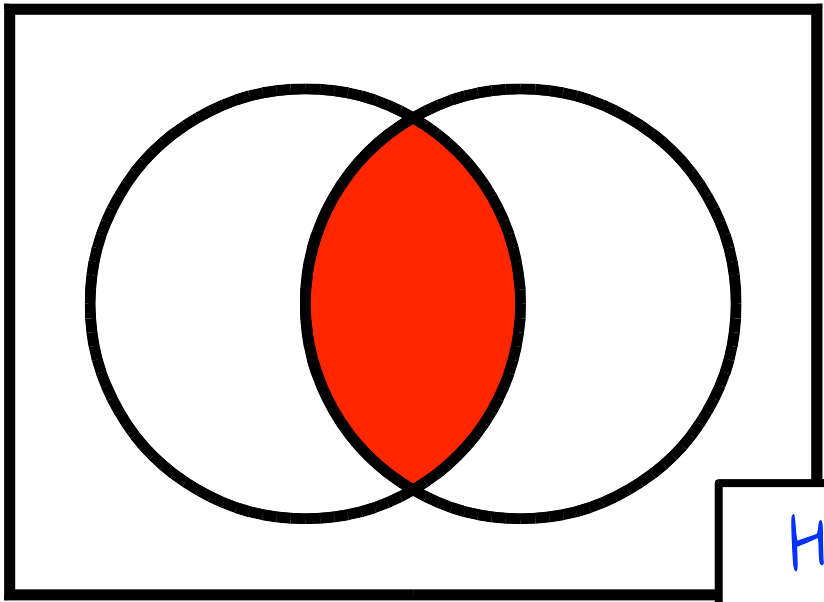


$$\overline{\overline{L_1} \cup \overline{L_2}}$$

Hey, it's De Morgan's laws!

Concatenation

String Concatenation

- If $w \in \Sigma^*$ and $x \in \Sigma^*$, the ***concatenation*** of w and x, denoted $\boldsymbol{wx}$, is the string formed by tacking all the characters of x onto the end of w.
- Example: if w = **quo** and x = **kka**, the concatenation wx = **quokka**.
- This is analogous to the + operator for strings in many programming languages.
- Some facts about concatenation:
  - The empty string ε is the ***identity element*** for concatenation:

$$\boldsymbol{w\varepsilon = \varepsilon w = w}$$

  - Concatenation is ***associative***:

$$\boldsymbol{wxy = w(xy) = (wx)y}$$

Concatenation

- The ***concatenation*** of two languages L_1 and L_2 over the alphabet Σ is the language

$$L_1L_2 = \{ x \mid \exists w_1 \in L_1.\ \exists w_2 \in L_2.\ x = w_1w_2 \}$$

- Let L_1 = { **ab**, **ba** } and L_2 = { **aa**, **bb** }. What is L_1L_2?

Answer at
https://cs103.stanford.edu/pollev

Concatenation Example

- Let Σ = { **a**, **b**, …, **z**, **A**, **B**, …, **Z** } and consider these languages over Σ:
  - ***Noun*** = { **Puppy**, **Rainbow**, **Whale**, … }
  - ***Verb*** = { **Hugs**, **Juggles**, **Loves**, … }
  - ***The*** = { **The** }
- The language ***TheNounVerbTheNoun*** is
  - { **ThePuppyHugsTheWhale**,
    TheWhaleLovesTheRainbow,
    TheRainbowJugglesTheRainbow, … }

Concatenation

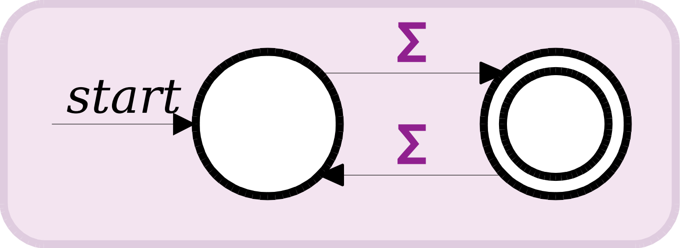
- The ***concatenation*** of two languages L_1 and L_2 over the alphabet Σ is the language

$$\boldsymbol{L_1L_2 = \{\ x \mid \exists w_1 \in L_1.\ \exists w_2 \in L_2.\ x = w_1w_2\ \}}$$

- Two views of L_1L_2:
  - The set of all strings that can be made by concatenating a string in L_1 with a string in L_2.
  - The set of strings that can be split into two pieces: a piece from L_1 and a piece from L_2.

$L_1 = \{ w \in \{\texttt{a}, \texttt{b}\}^* \mid w$ has *odd* length $\}$
$L_2 = \{ w \in \{\texttt{a}, \texttt{b}\}^* \mid w$ has length exactly three $\}$

Construct an NFA for L_1L_2.



DFA for L_1

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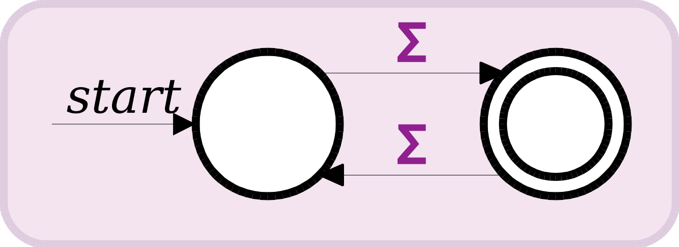

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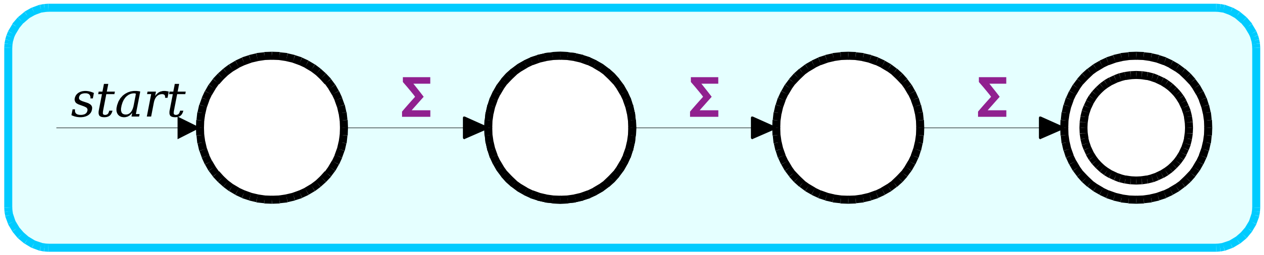

NFA for L_2

L_1 = { w ∈ {a, b}* | w has *odd* length }
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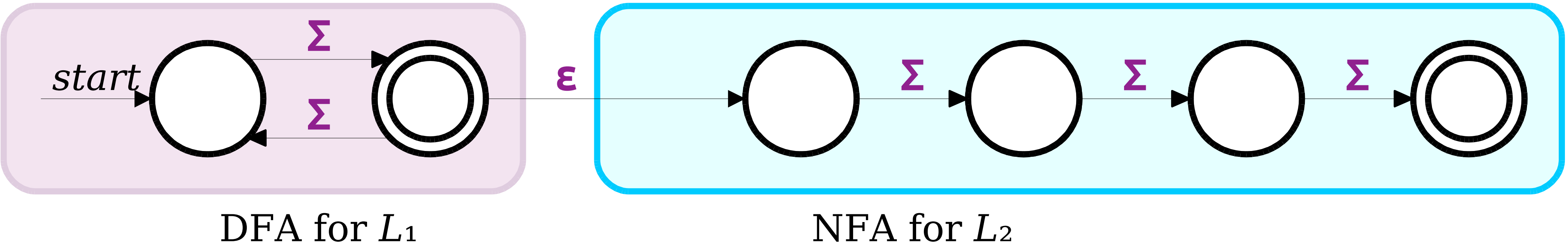

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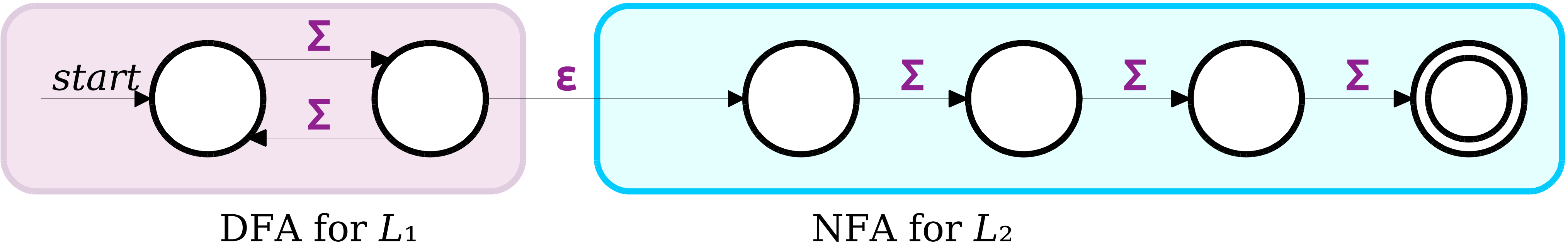

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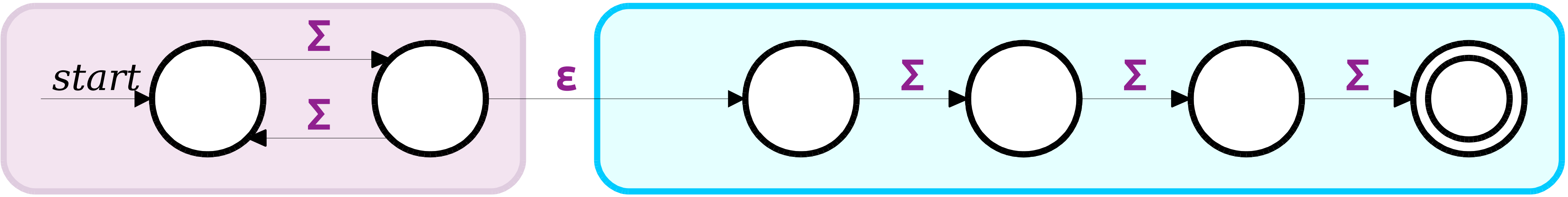

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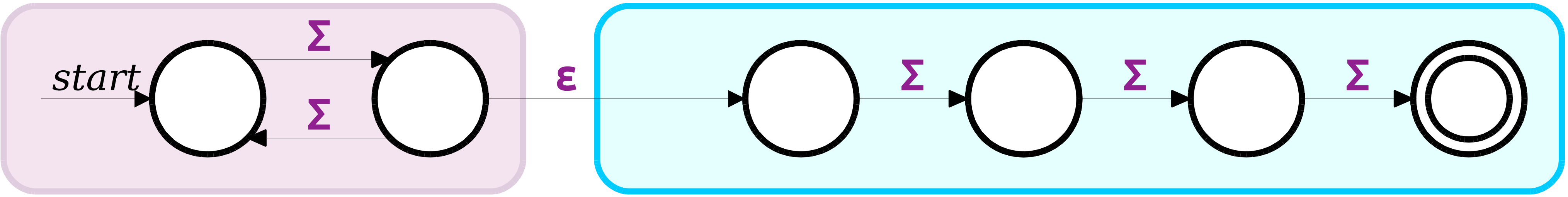

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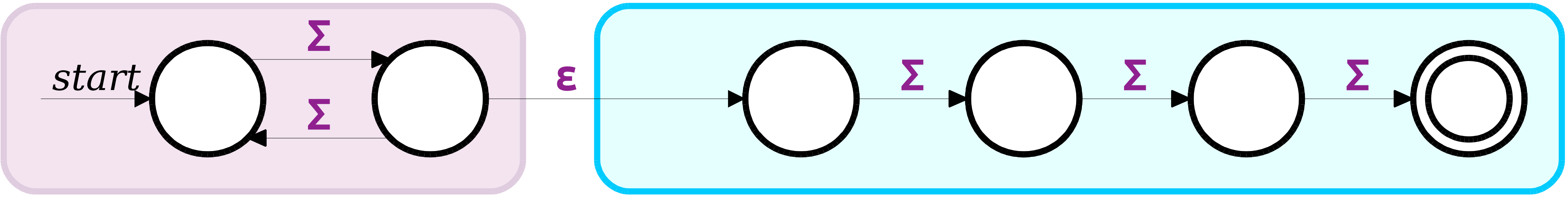
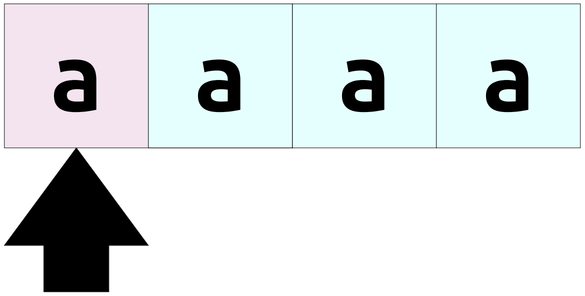


DFA for L_1 NFA for L_2



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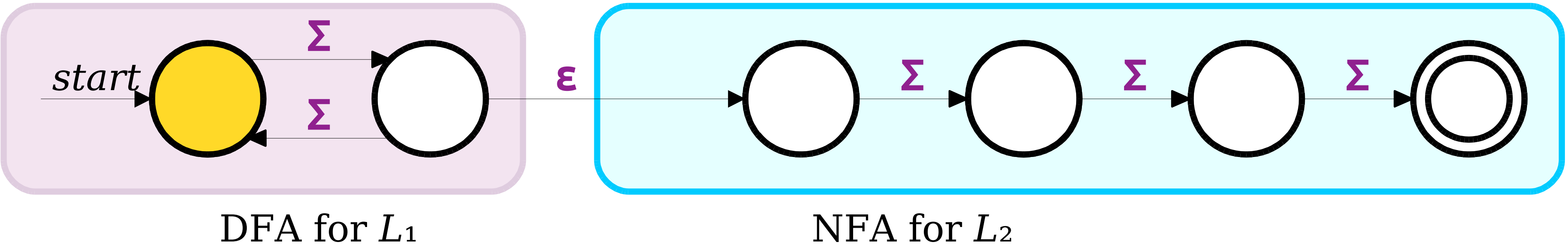

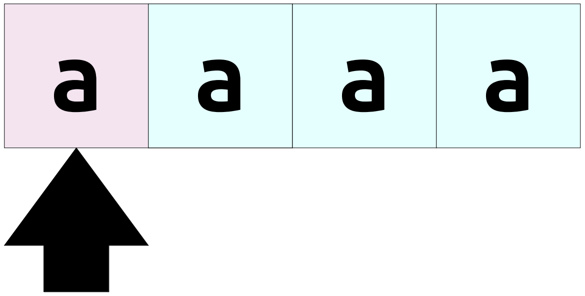

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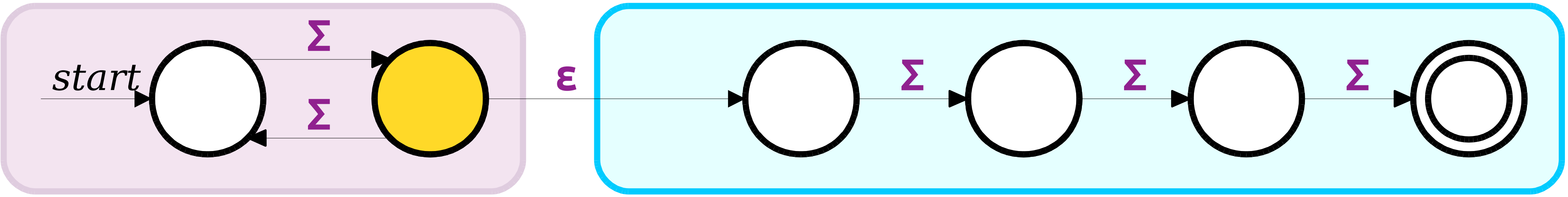

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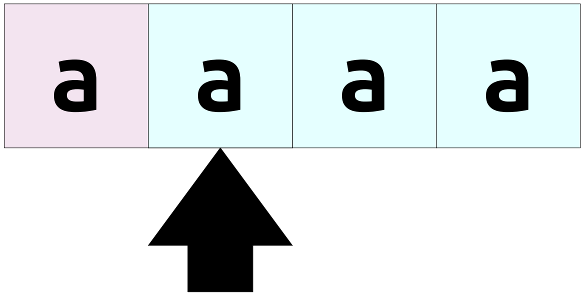

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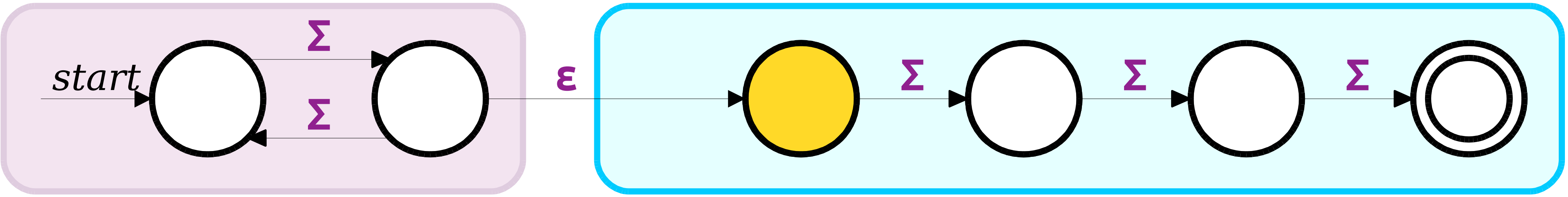

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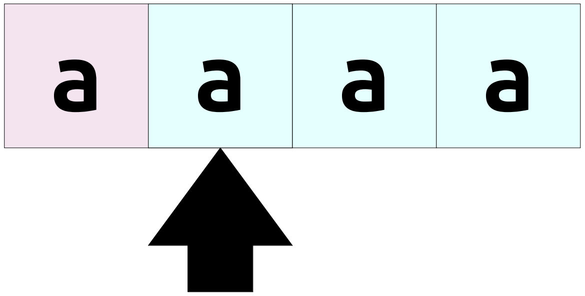

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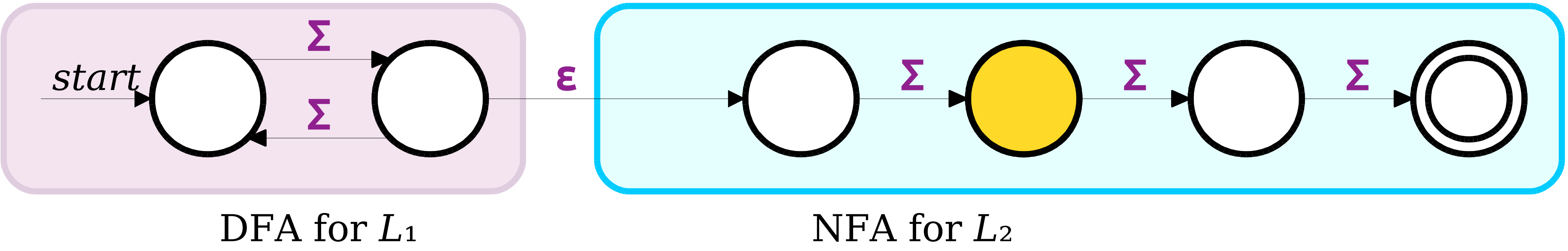

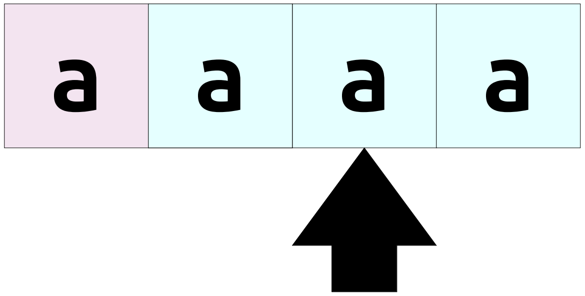

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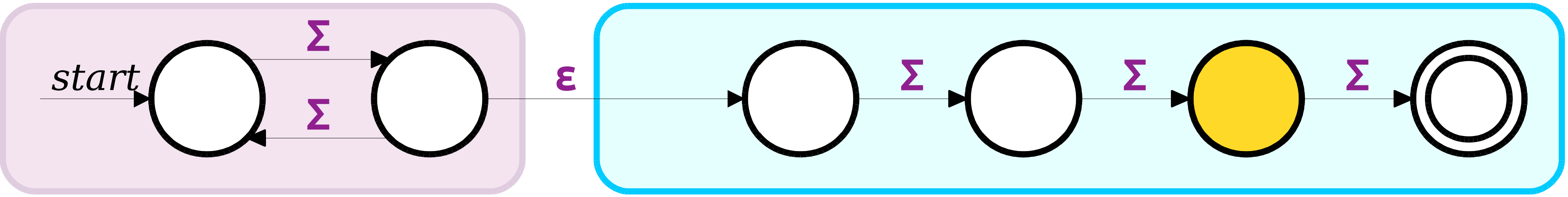

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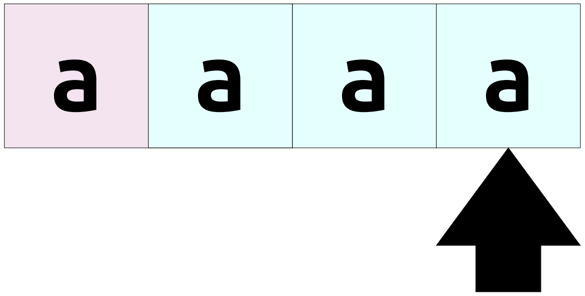

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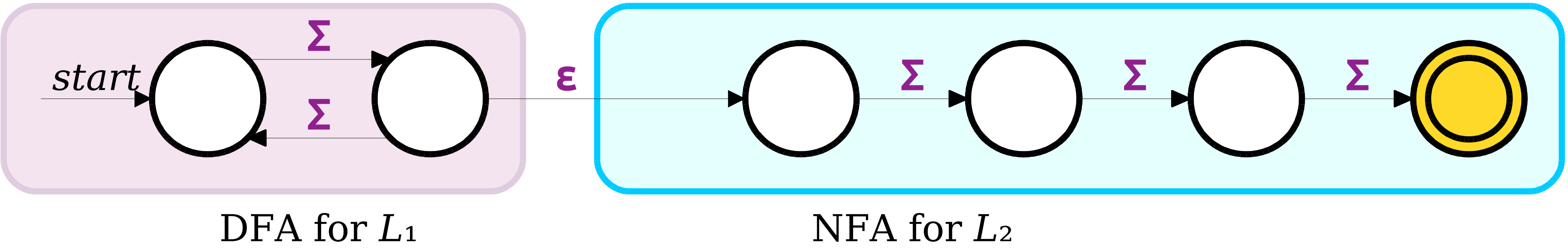

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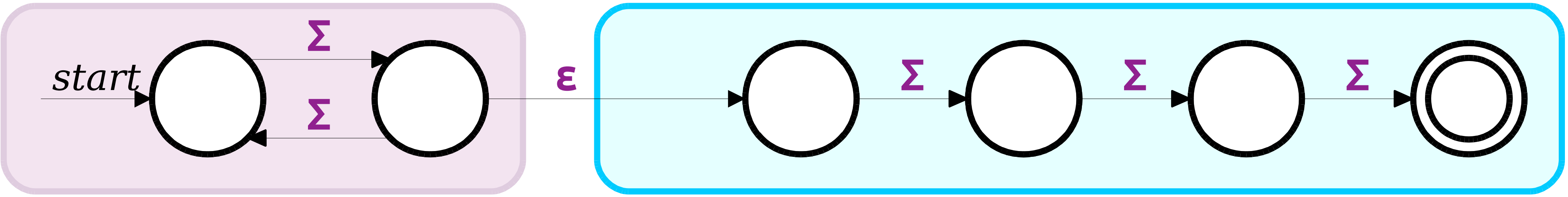

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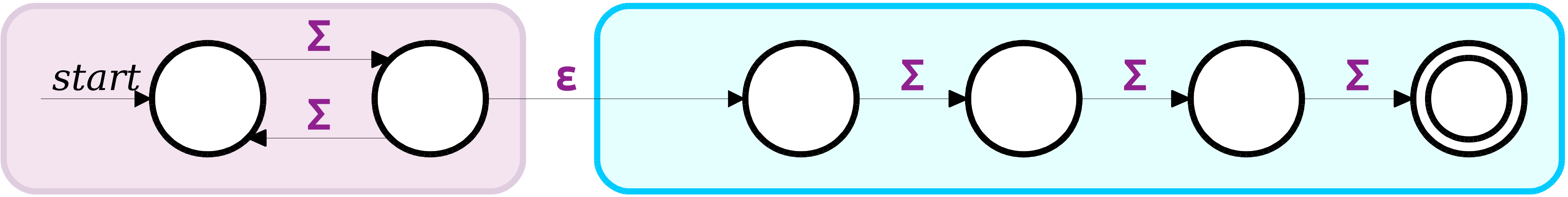

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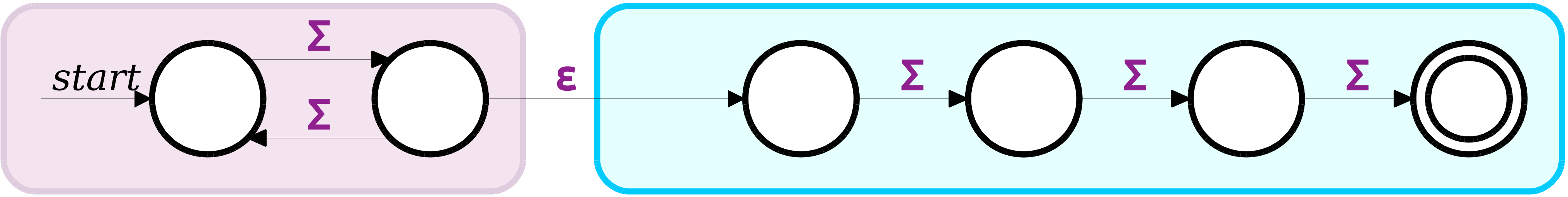

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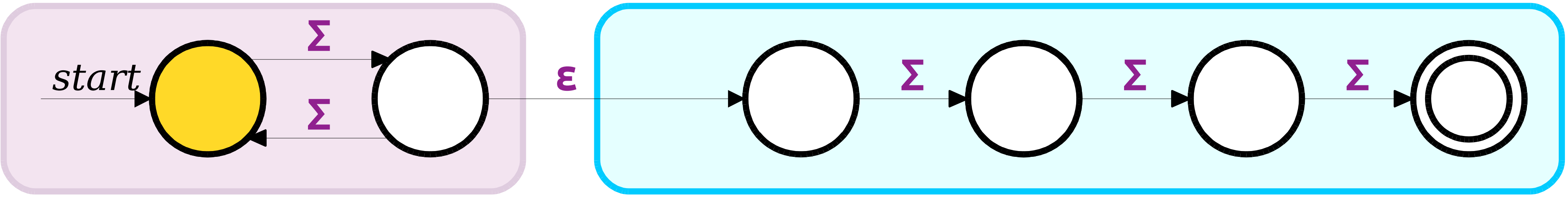

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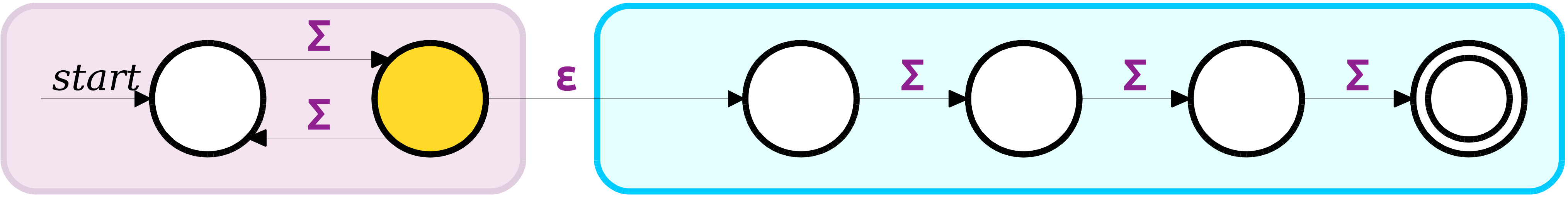

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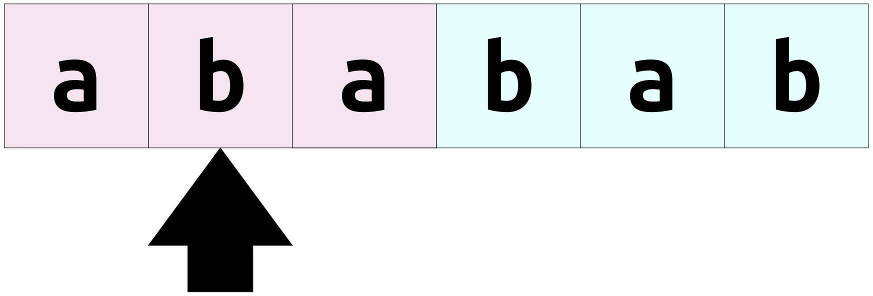

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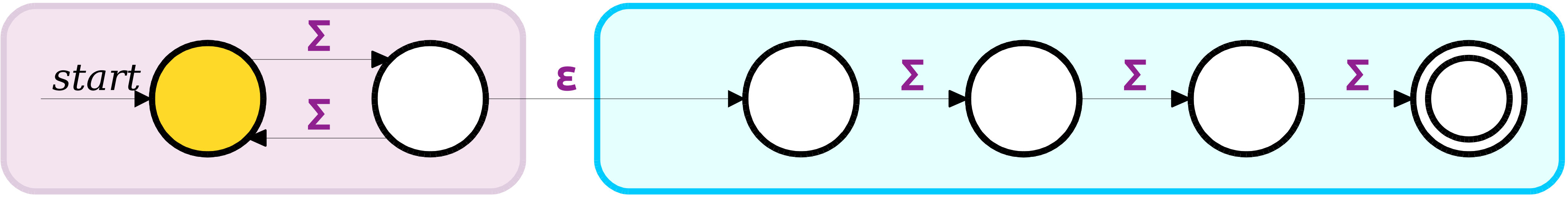

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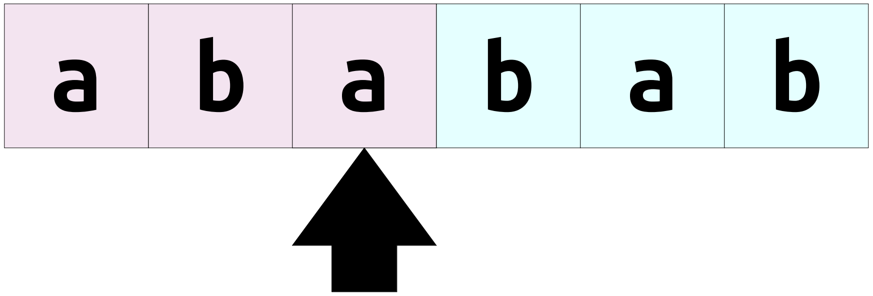

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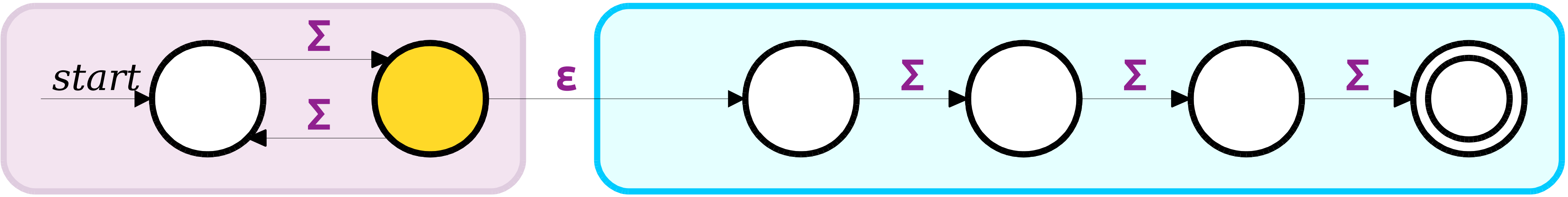

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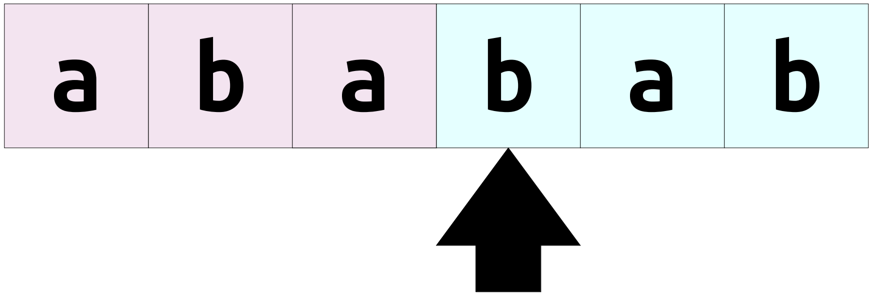

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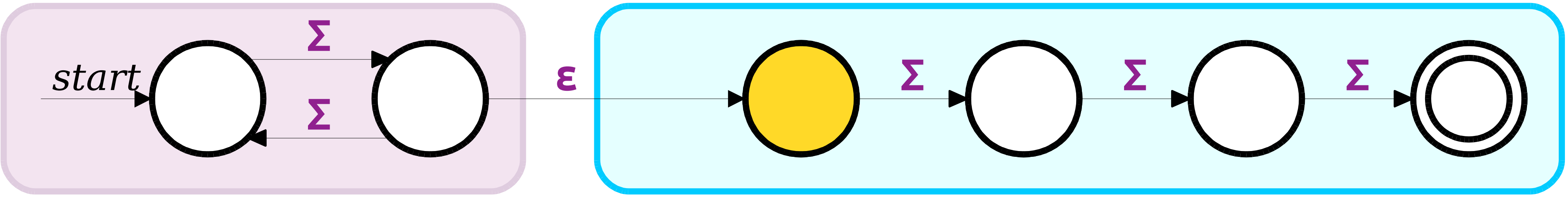

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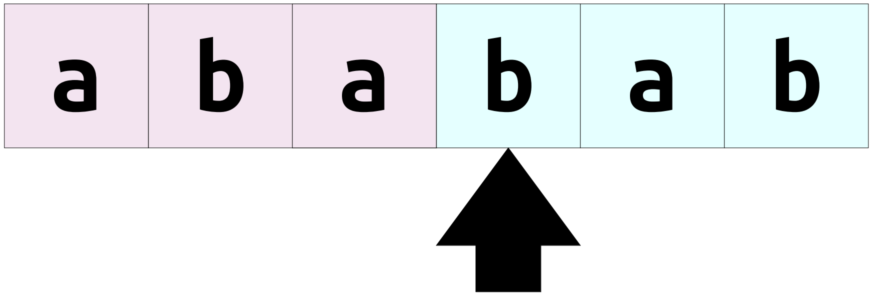

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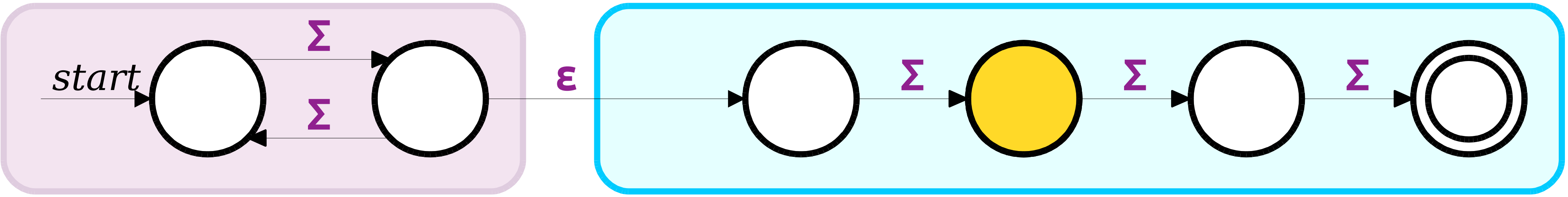

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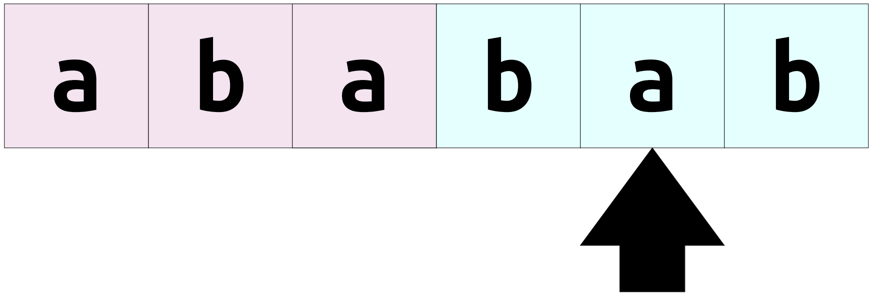

a	b	a	b	a	b

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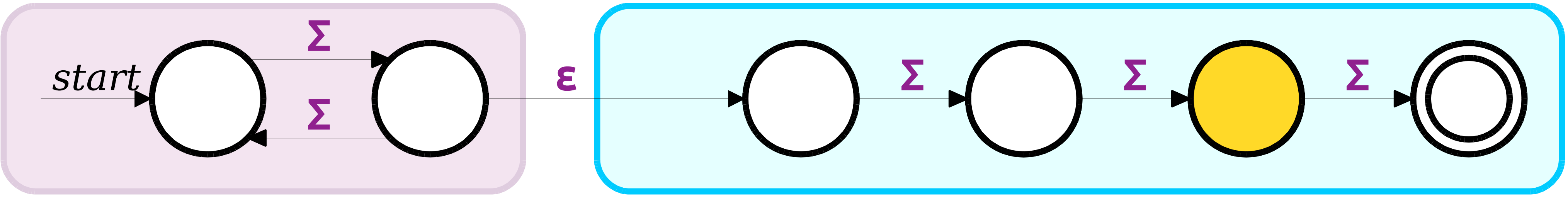

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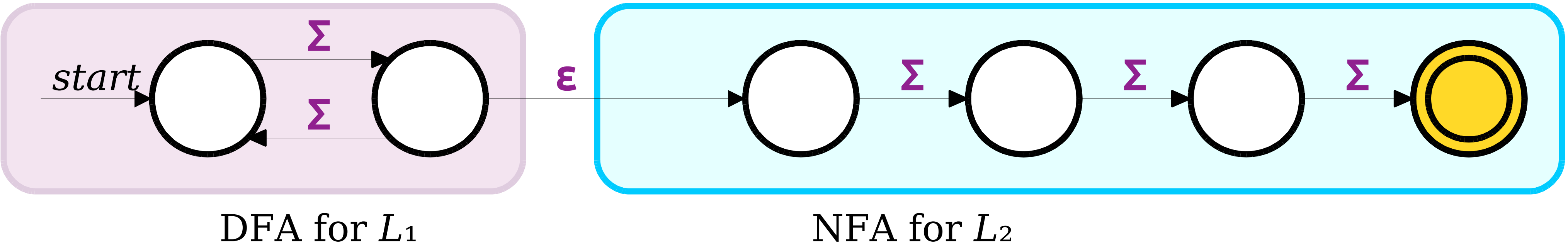

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Construct an NFA for L_1L_2.

The Kleene Star

Lots and Lots of Concatenation

- Consider the language L = { **aa**, **b** }
- LL is the set of strings formed by concatenating pairs of strings in L.

{ **aaaa**, **aab**, **baa**, **bb** }

- LLL is the set of strings formed by concatenating triples of strings in L.

{ **aaaaaa**, **aaaab**, **aabaa**, **aabb**, **baaaa**, **baab**, **bbaa**, **bbb**}

- $LLLL$ is the set of strings formed by concatenating quadruples of strings in L.

{ **aaaaaaaa**, **aaaaaab**, **aaaabaa**, **aaaabb**, **aabaaaa**, **aabaab**, **aabbaa**, **aabbb**, **baaaaaa**, **baaaab**, **baabaa**, **baabb**, **bbaaaa**, **bbaab**, **bbbaa**, **bbbb**}

Language Exponentiation

- We can define what it means to “exponentiate” a language as follows:
- $L^0 = \{\varepsilon\}$
  - Intuition: The only string you can form by gluing no strings together is the empty string.
  - Notice that $\{\varepsilon\} \neq \emptyset$. Can you explain why?
- $L^{n+1} = LL^n$
  - Idea: Concatenating (n+1) strings together works by concatenating n strings, then concatenating one more.
- ***Question to ponder:*** Why define $L^0 = \{\varepsilon\}$?
- ***Question to ponder:*** What is $\emptyset^0$?

The Kleene Star

The Kleene Closure

- An important operation on languages is the ***Kleene closure***, or ***Kleene star***, which is defined as

$$L^* = \{\ w \in \Sigma^* \ \mid\ \exists n \in \mathbb{N}.\ w \in L^n\ \}$$

- Mathematically:

$$w \in L^* \quad \leftrightarrow \quad \exists n \in \mathbb{N}.\ w \in L^n$$

- Intuitively, L^* is the language all possible ways of concatenating zero or more strings in L together, possibly with repetition.
- ***Question to ponder:*** What is Ø*?

The Kleene Closure

If L = { **a**, **bb** }, then $L*$ = {

ε,

a, **bb**,

aa, **abb**, **bba**, **bbbb**,

aaa, **aabb**, **abba**, **abbbb**, **bbaa**, **bbabb**, **bbbba**, **bbbbbb**,

…

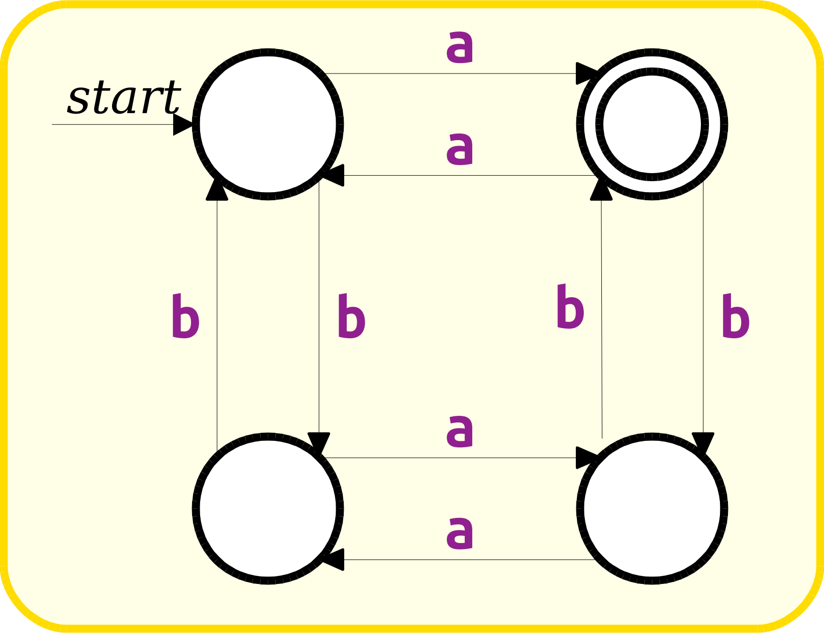
}

Think of L* as the set of strings you can make if you have a collection of stamps – one for each string in L – and you form every possible string that can be made from those stamps.

Idea: Can we convert an NFA for a language L to an NFA for language L^*?

$L = \{\ w \in \{\texttt{a}, \texttt{b}\}^* \mid w$ has an odd number of $\texttt{a}$'s and an even number of $\texttt{b}$'s $\}$

Construct an NFA for L^*.



DFA for L

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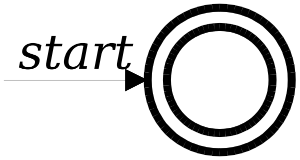

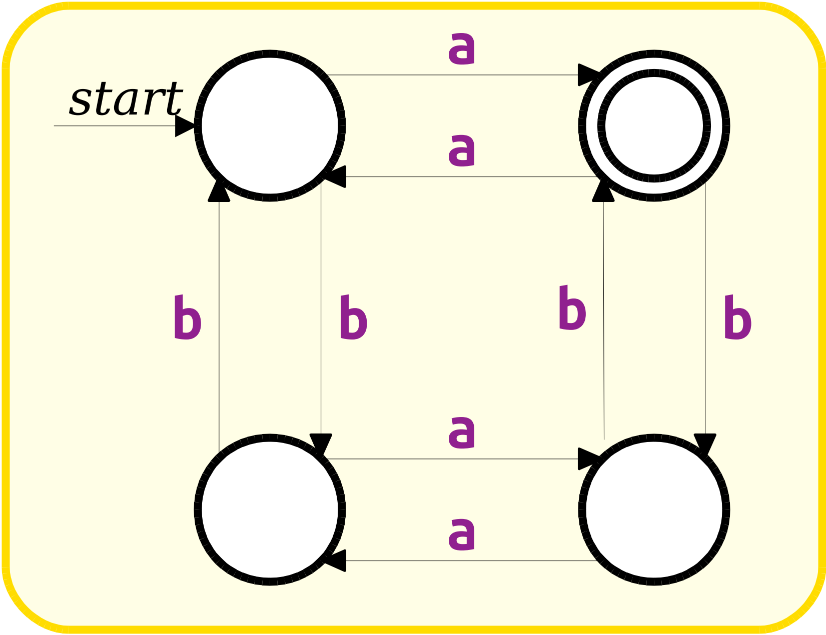

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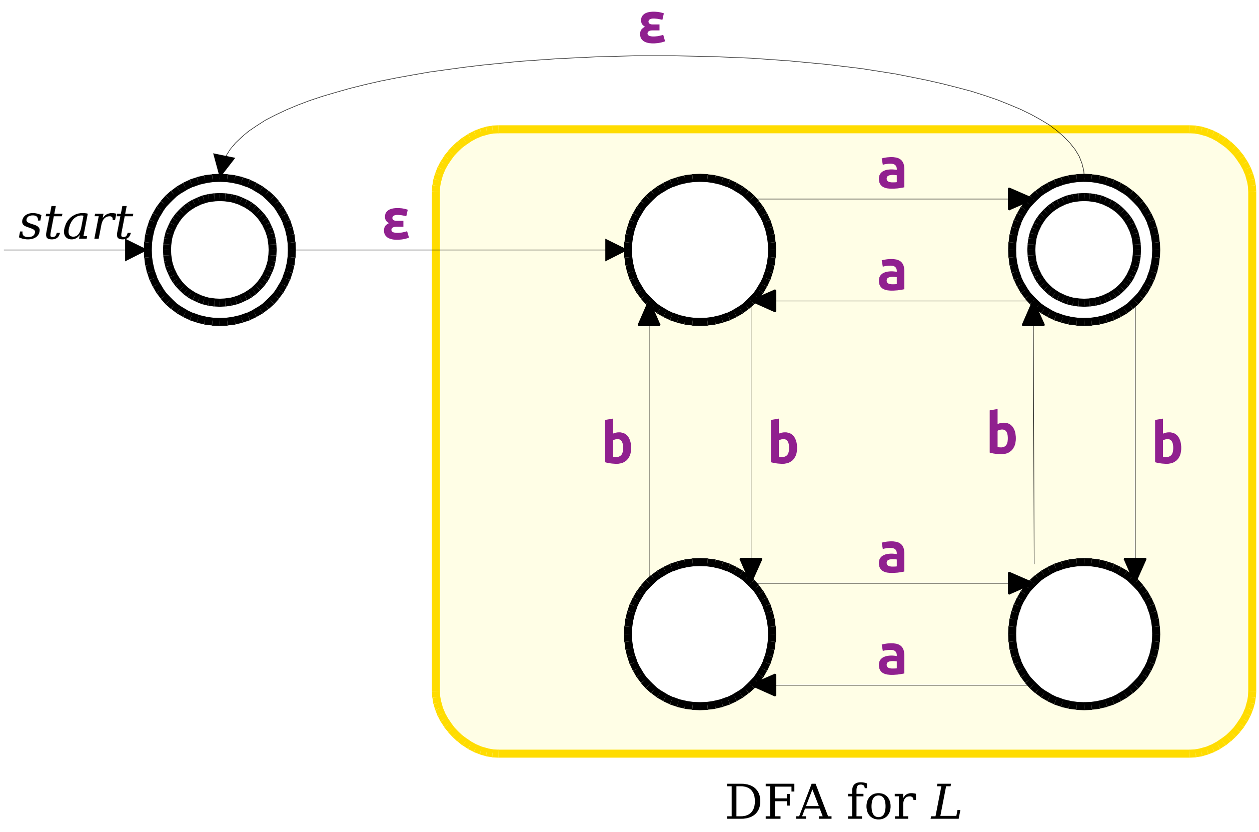

L = { $w \in \{$a, b$\}^*$ | w has an odd number of a's and an even number of b's }

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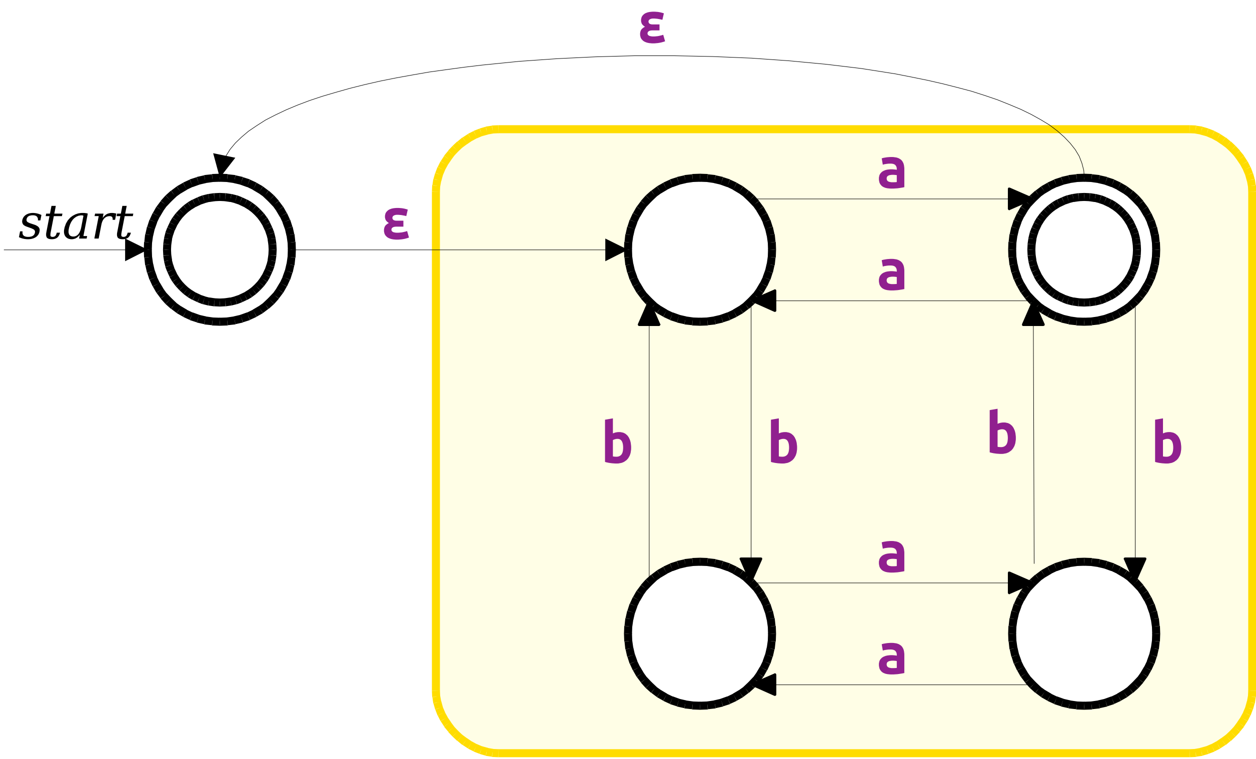

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a	b	b	a	b	a	b	a	b	a	b

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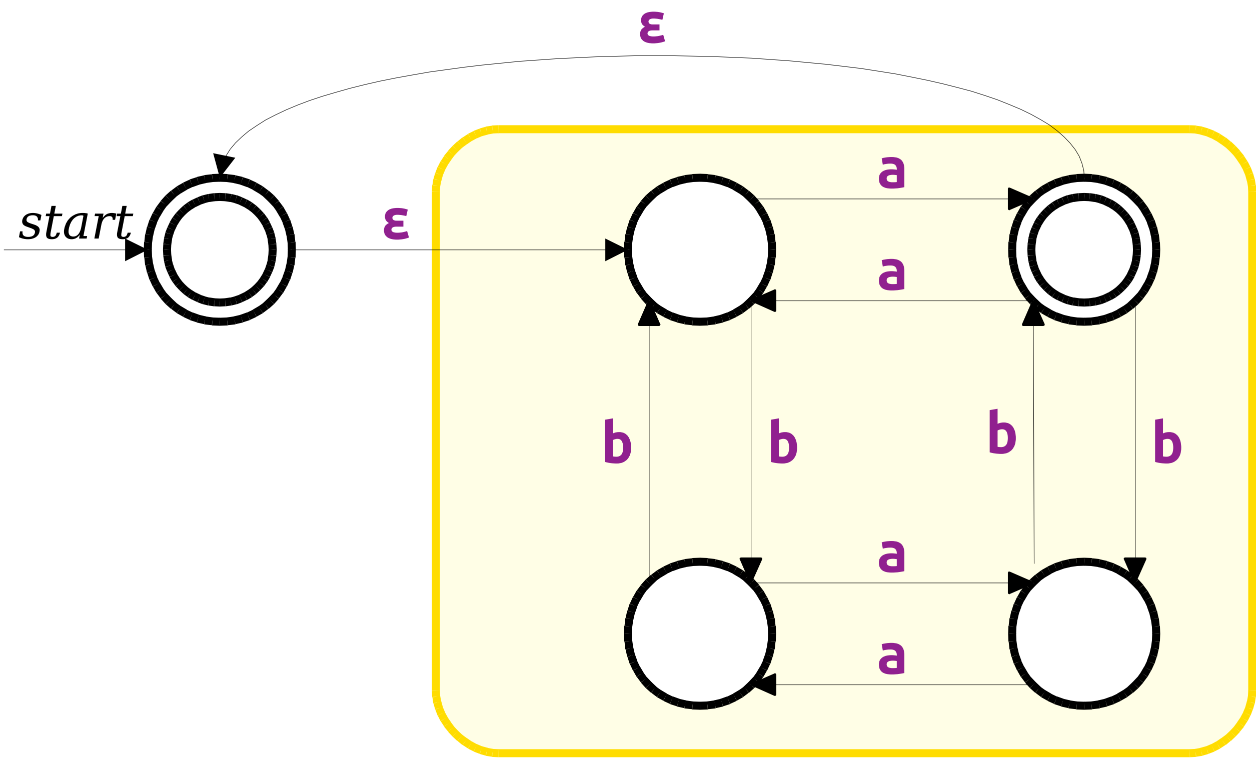

DFA for *L*

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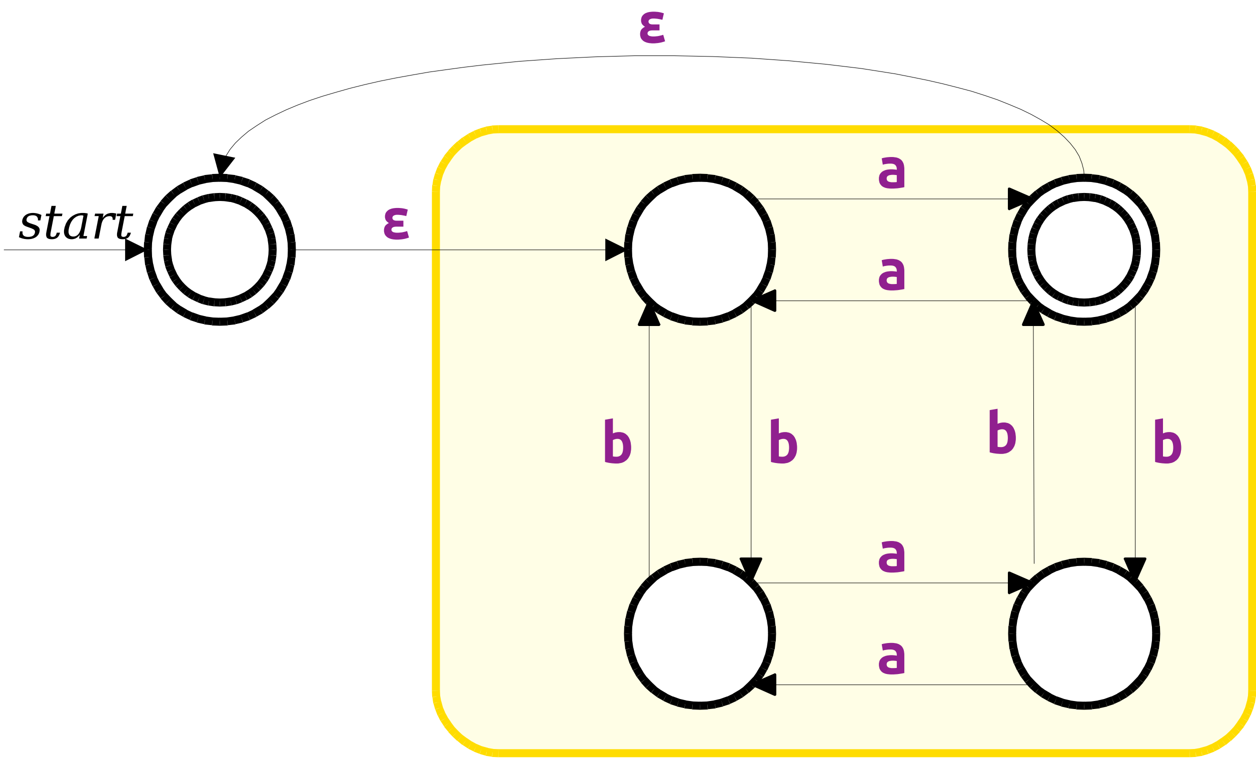

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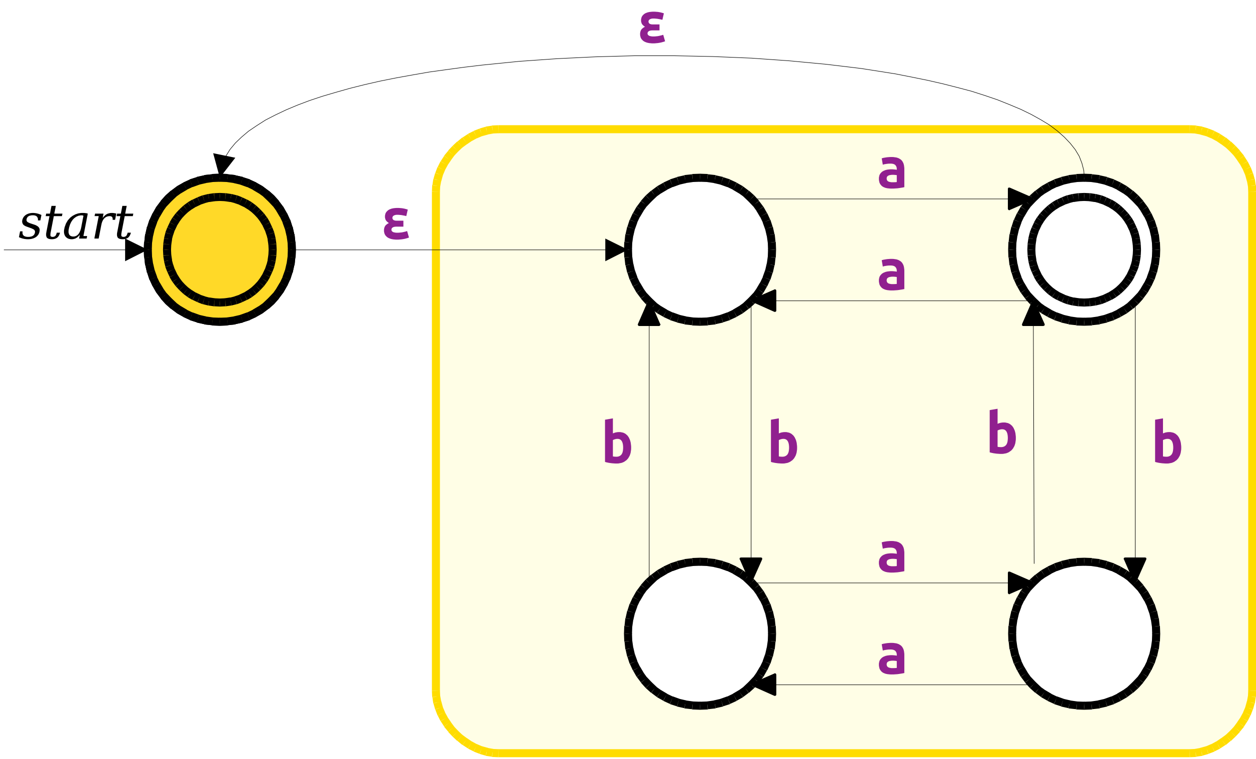

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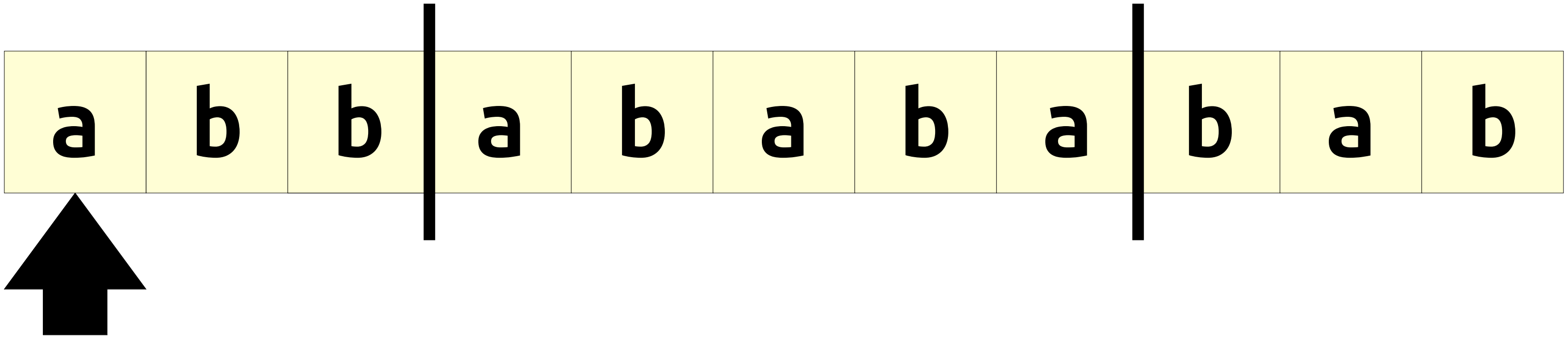

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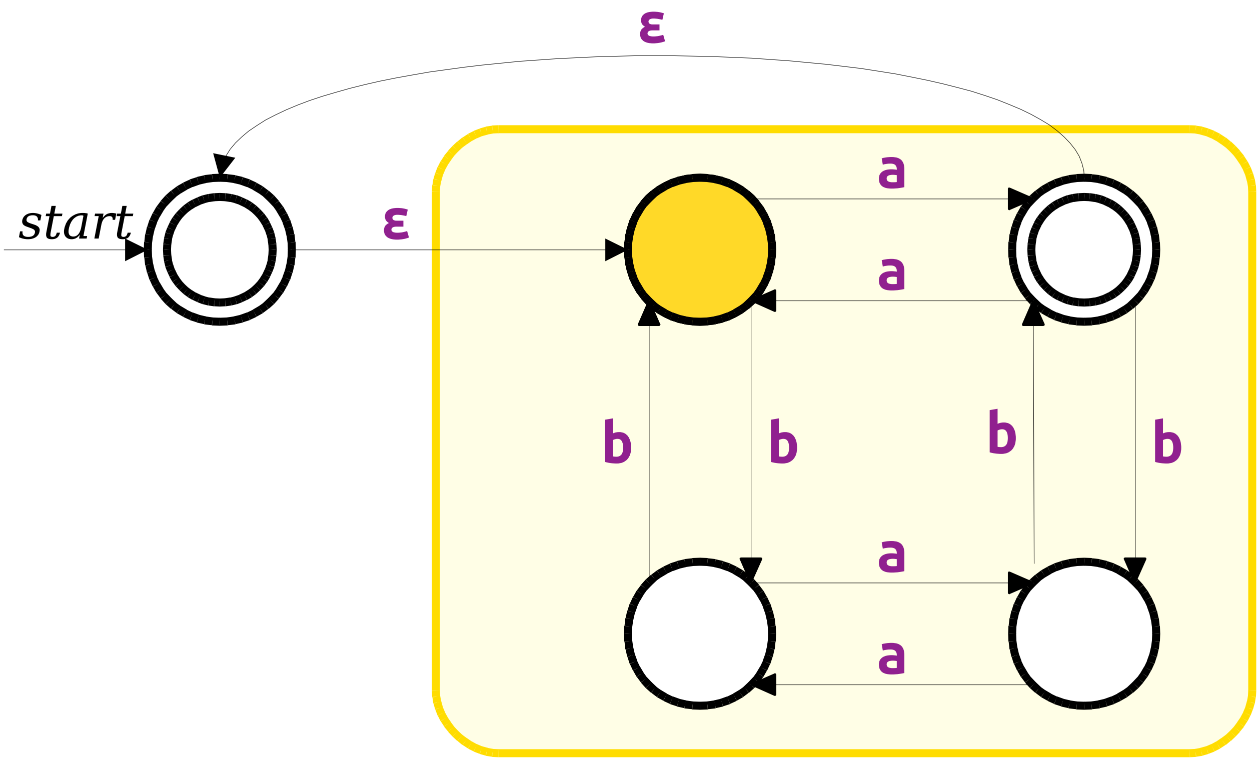

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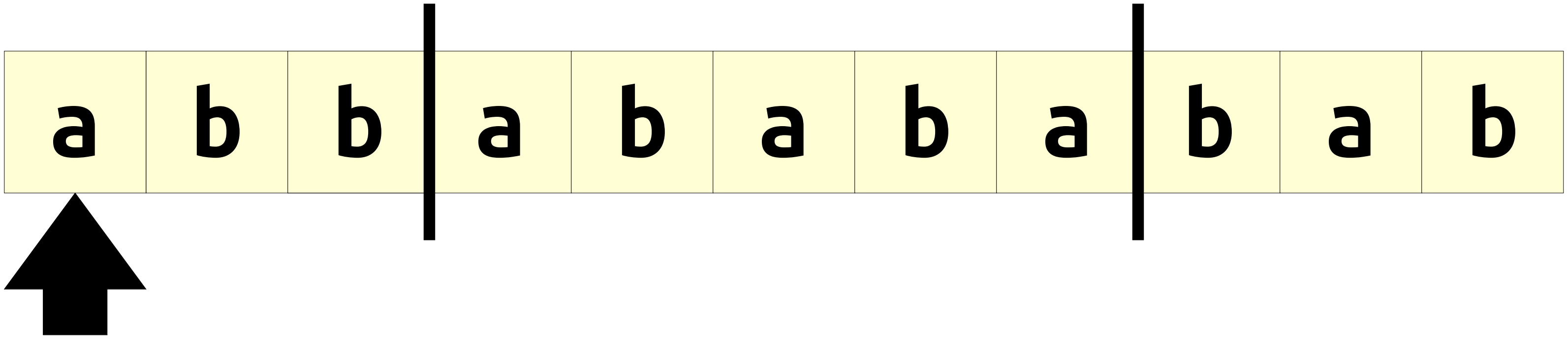

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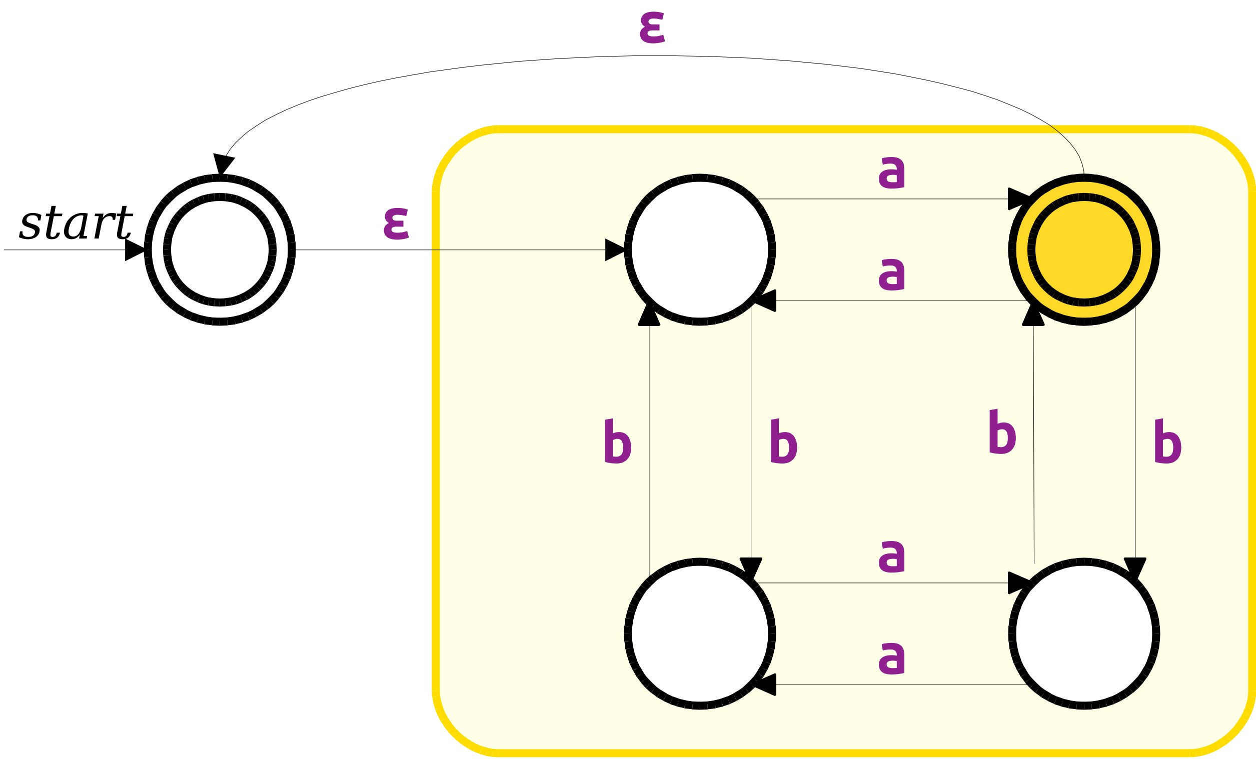

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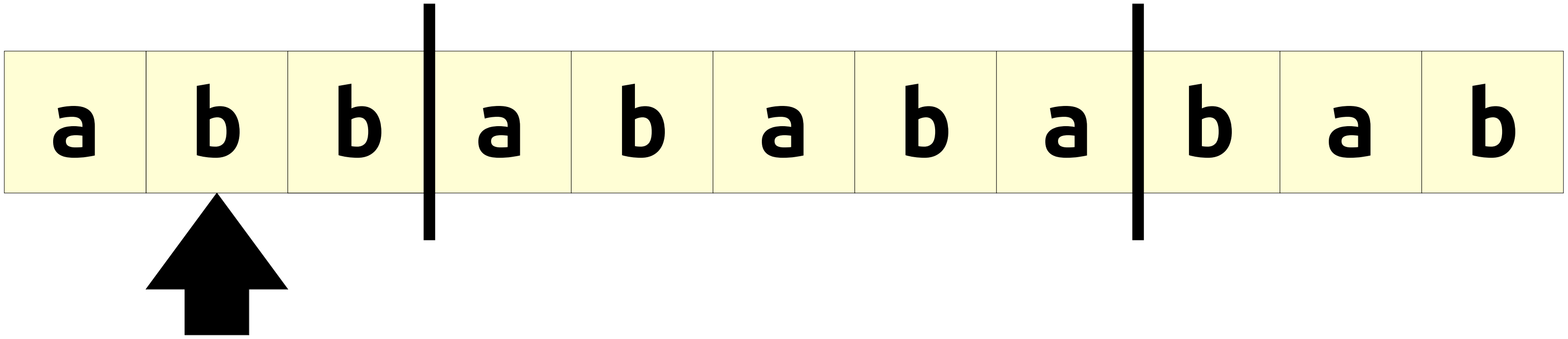

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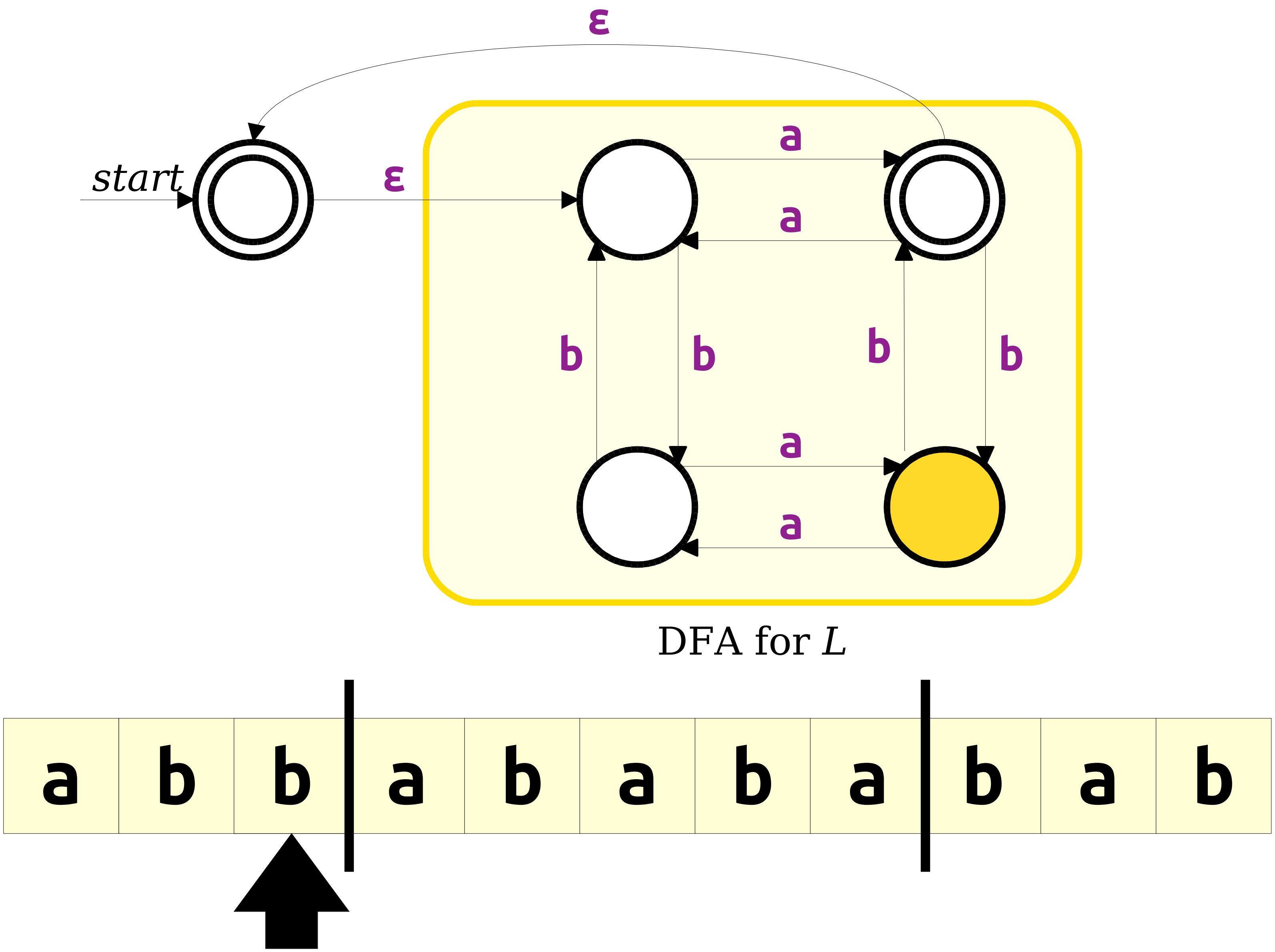

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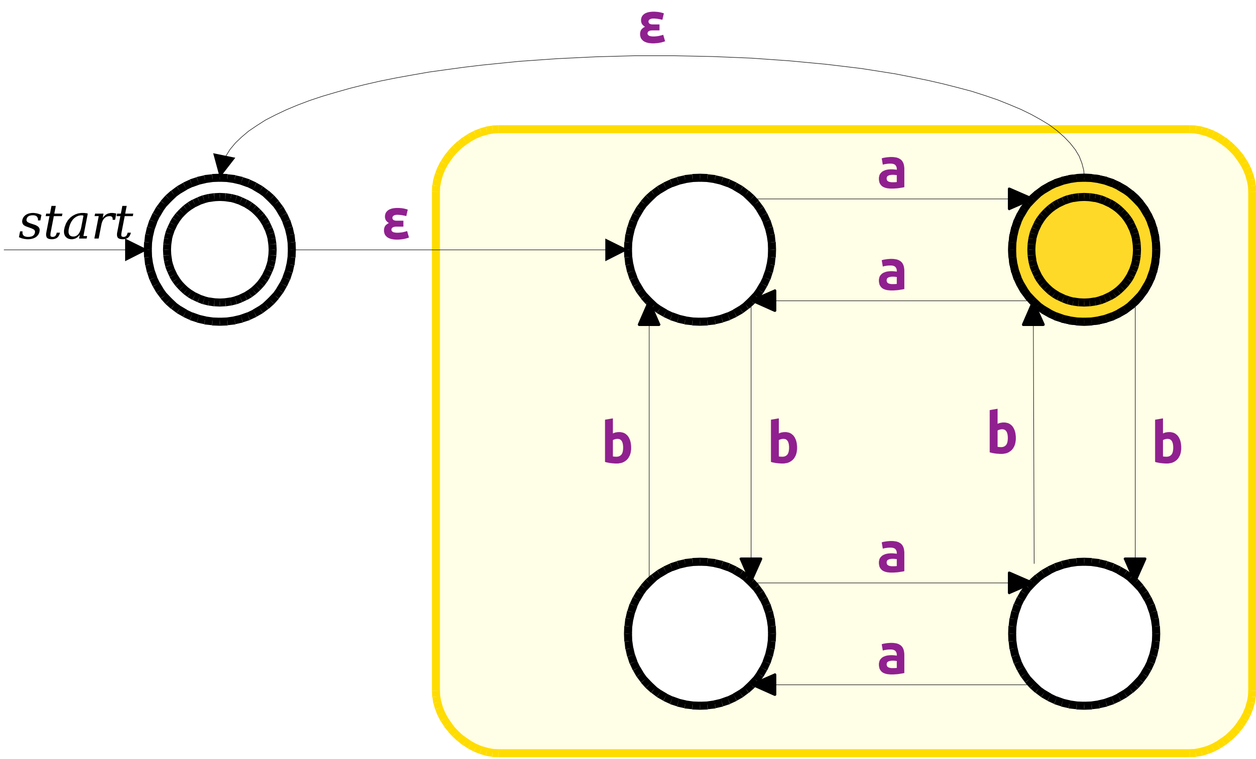

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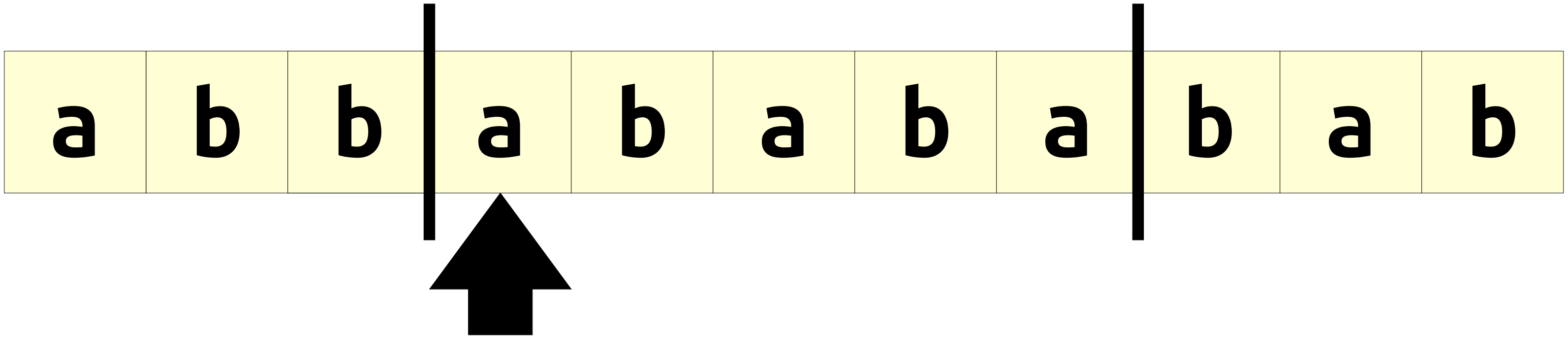

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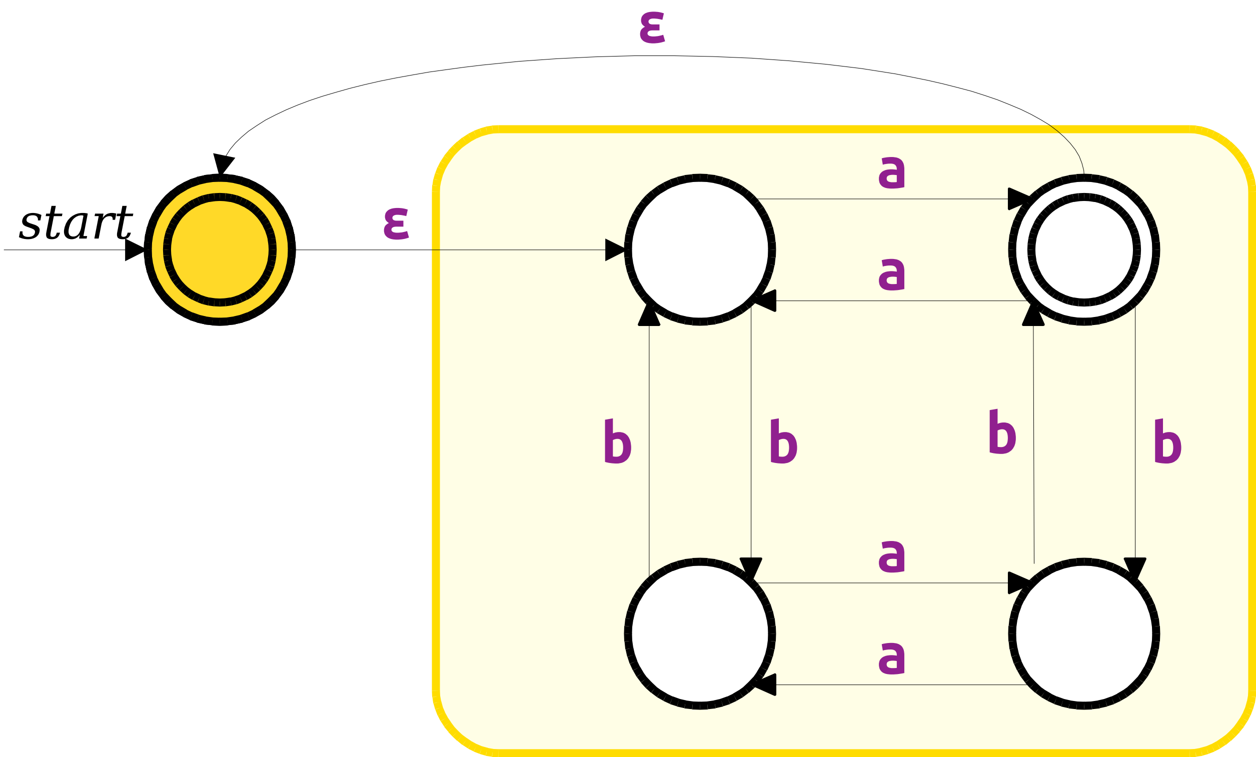

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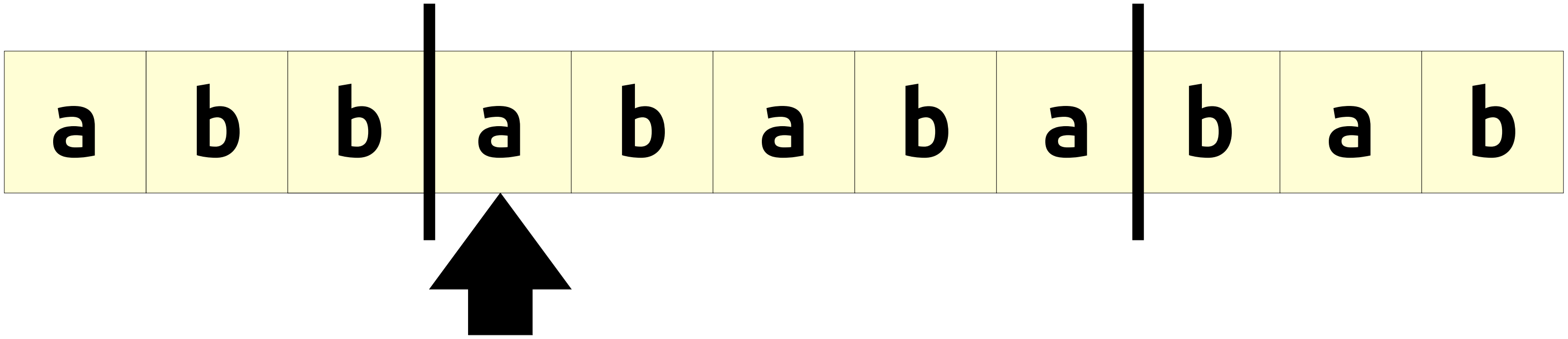

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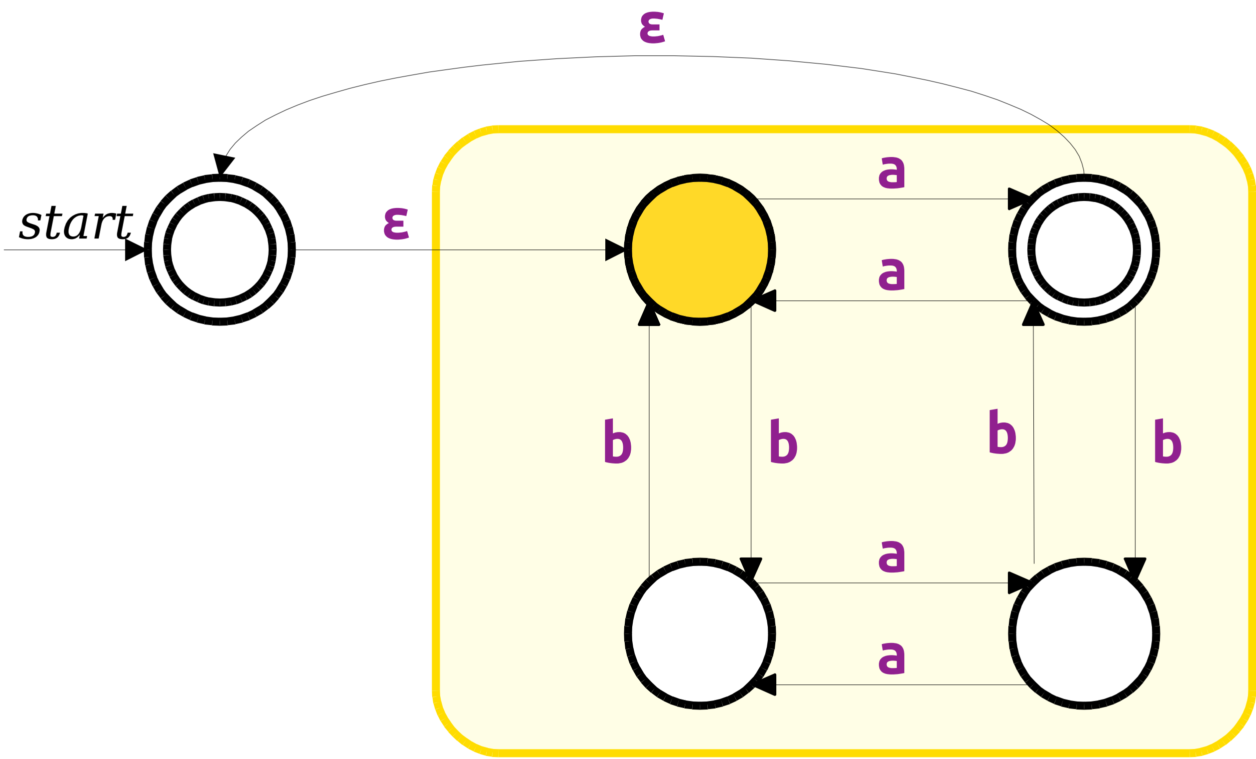

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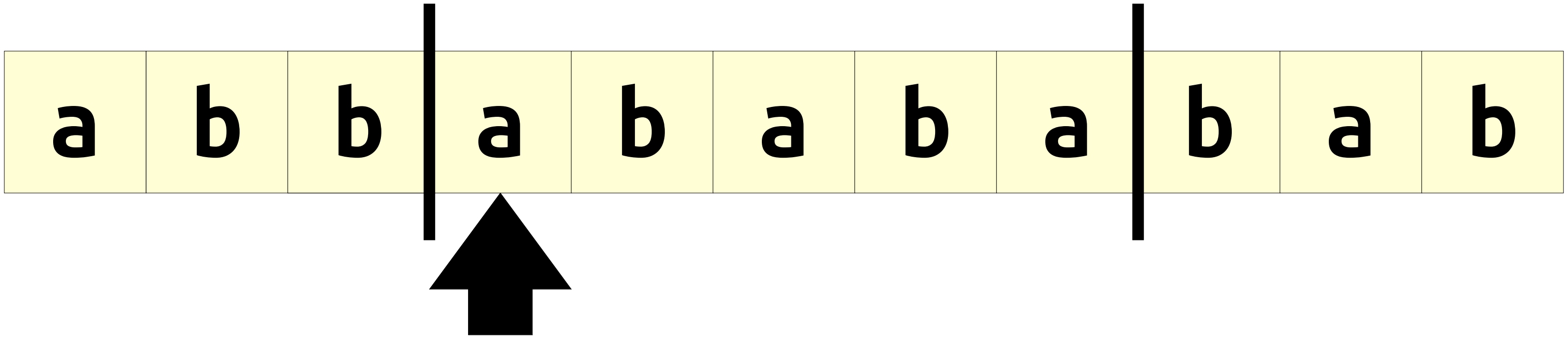

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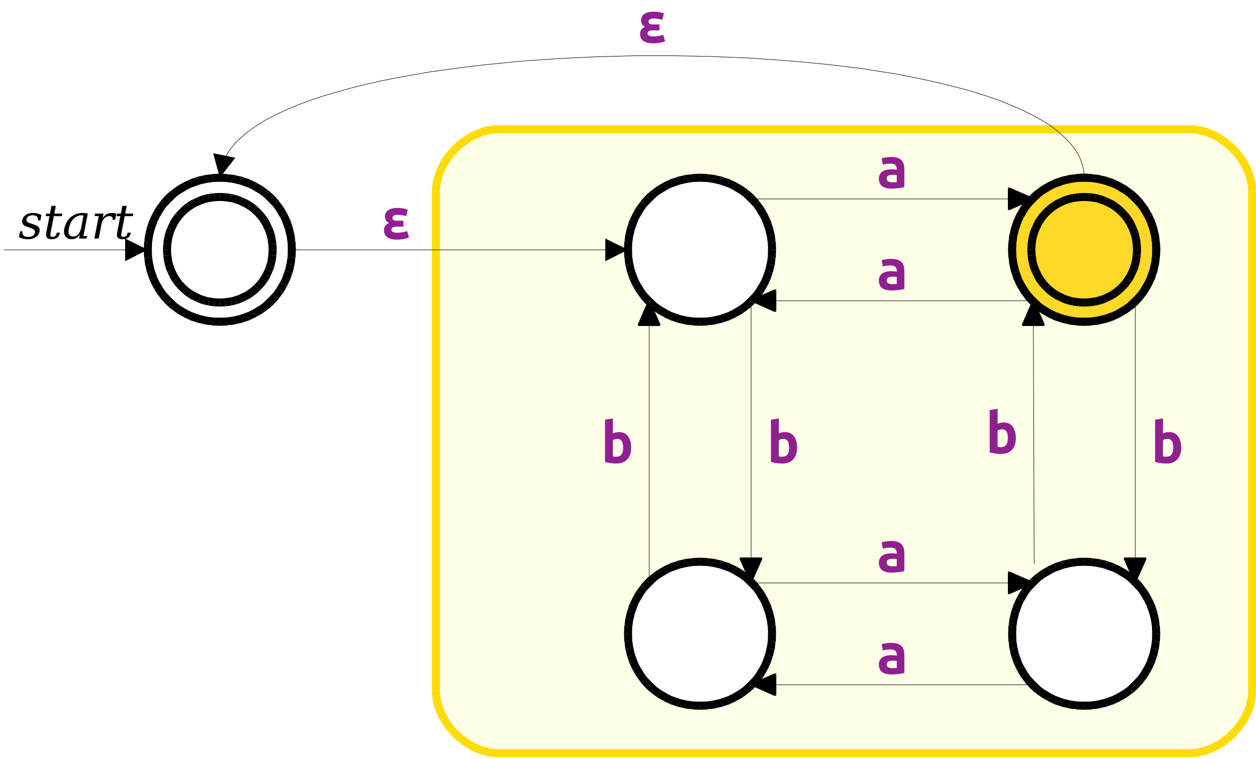

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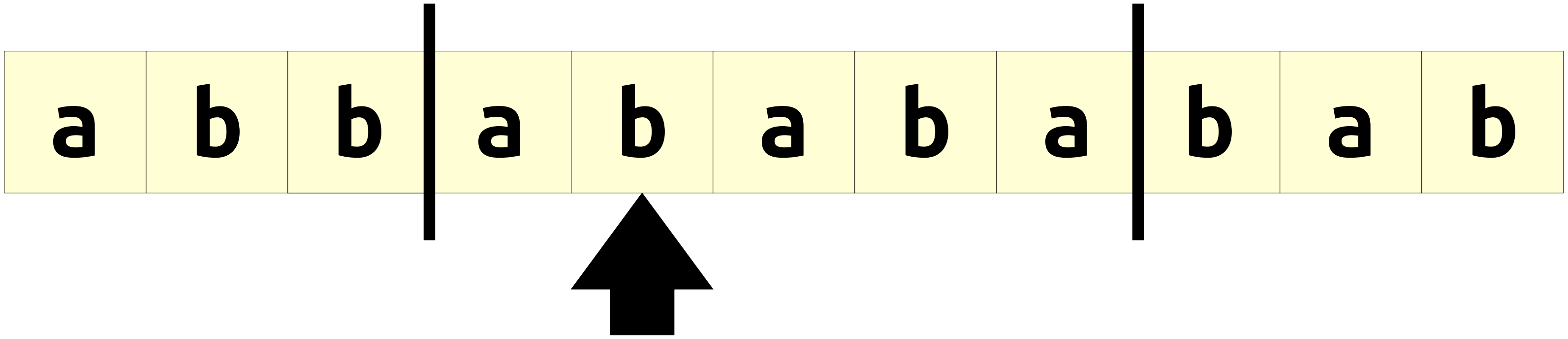

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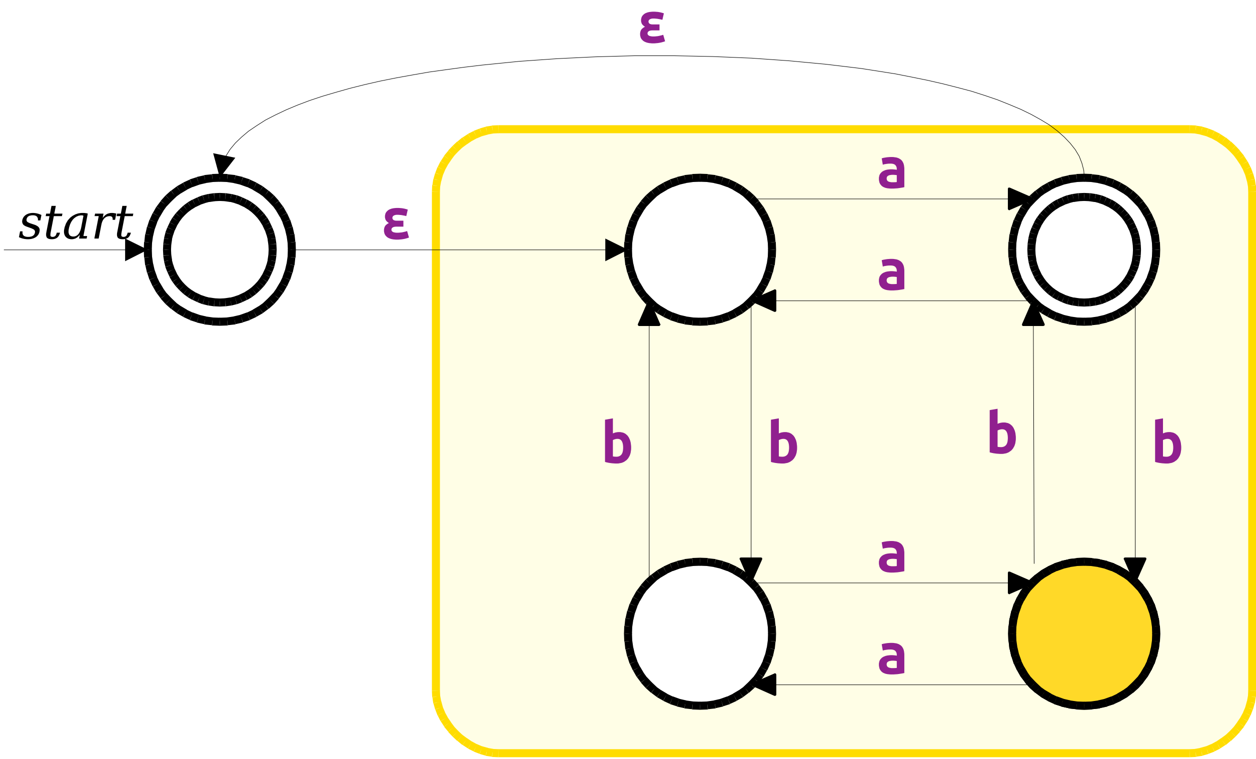

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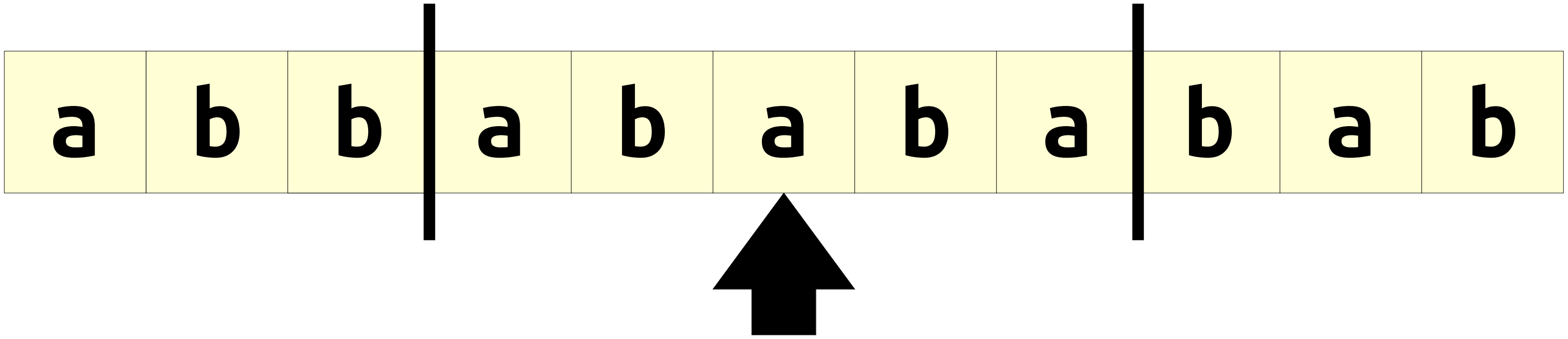

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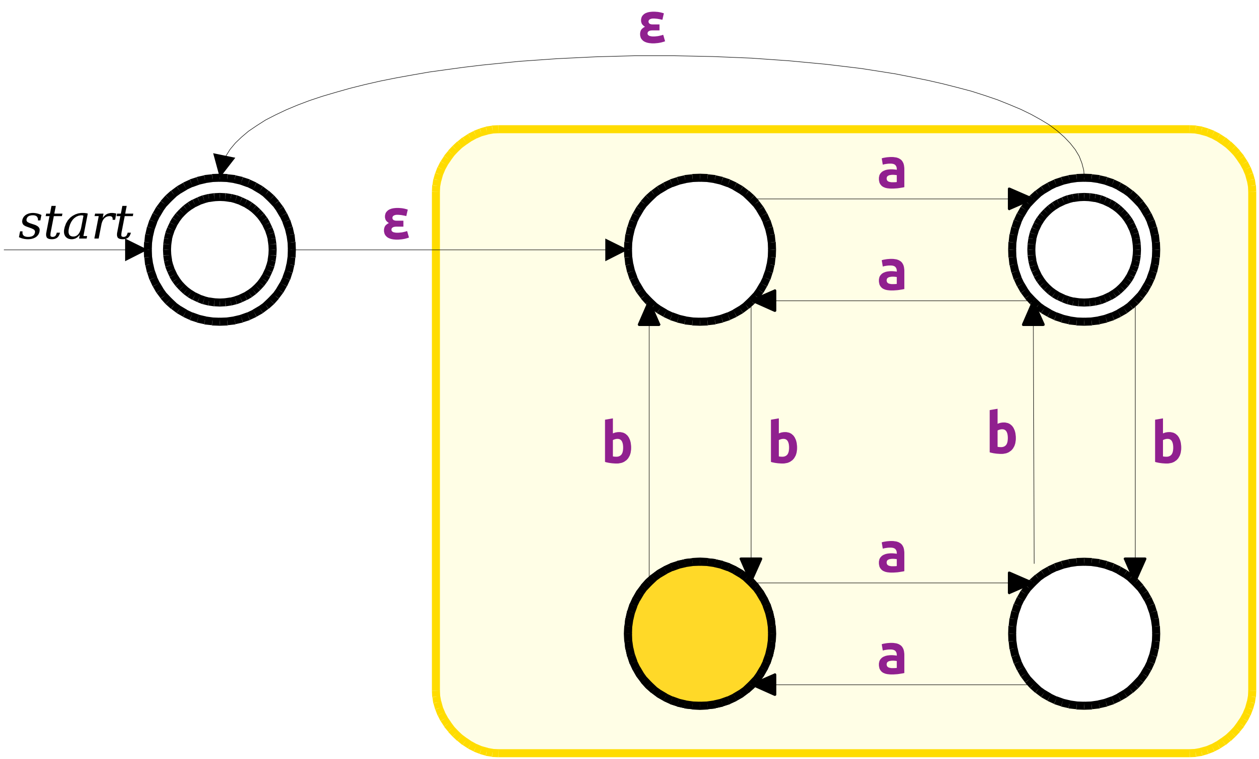

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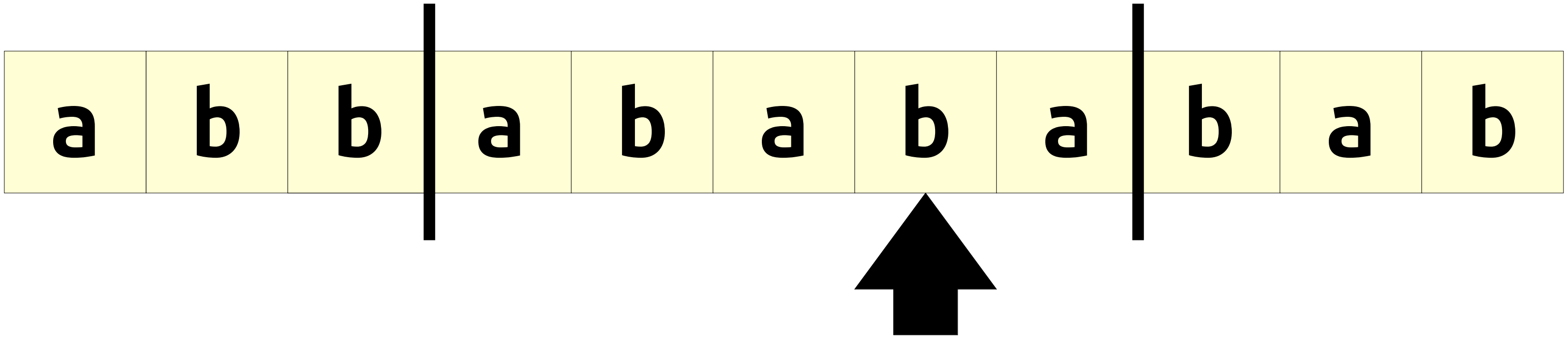

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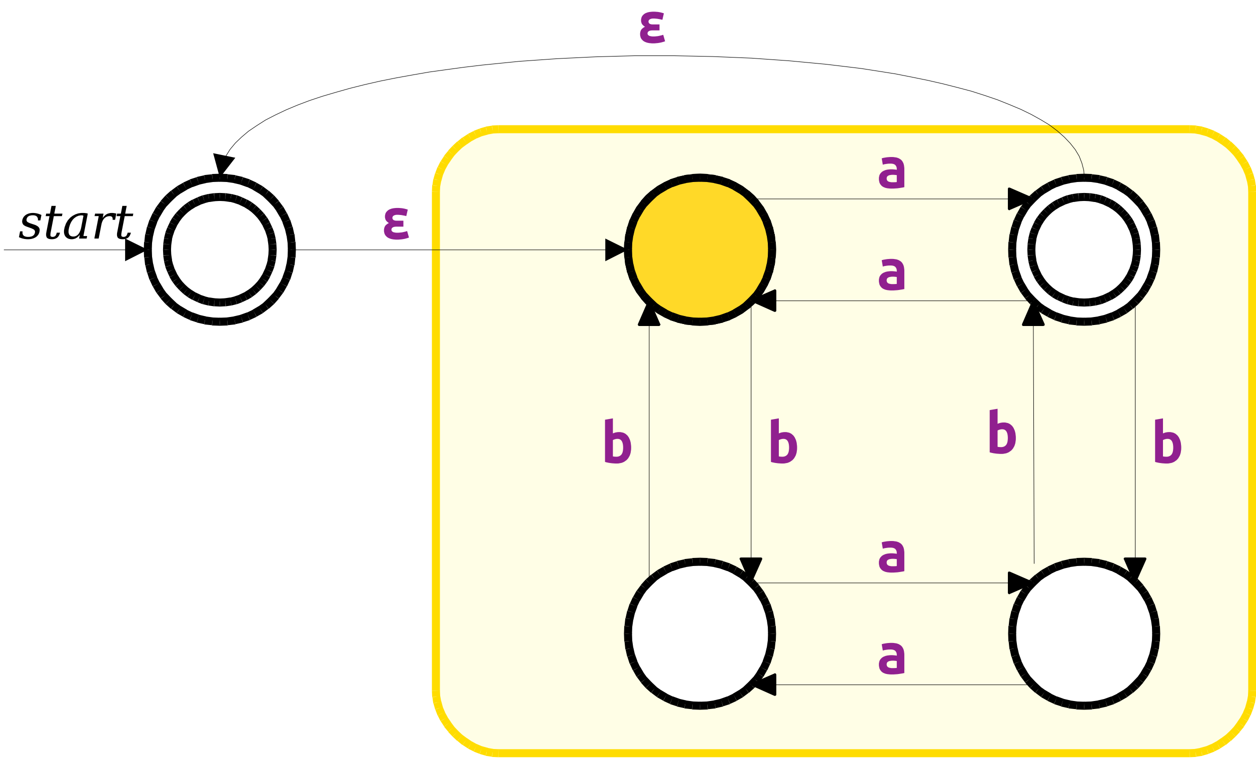

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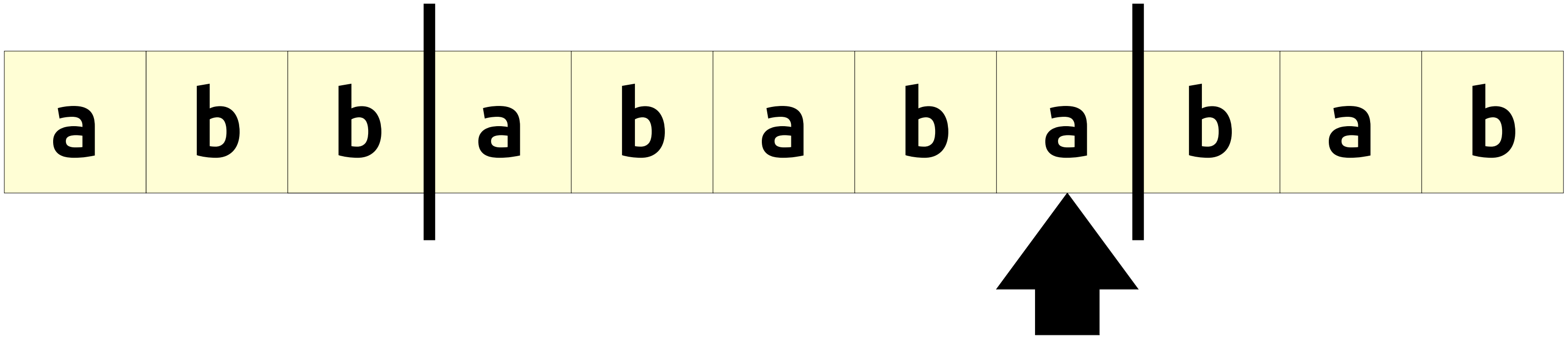

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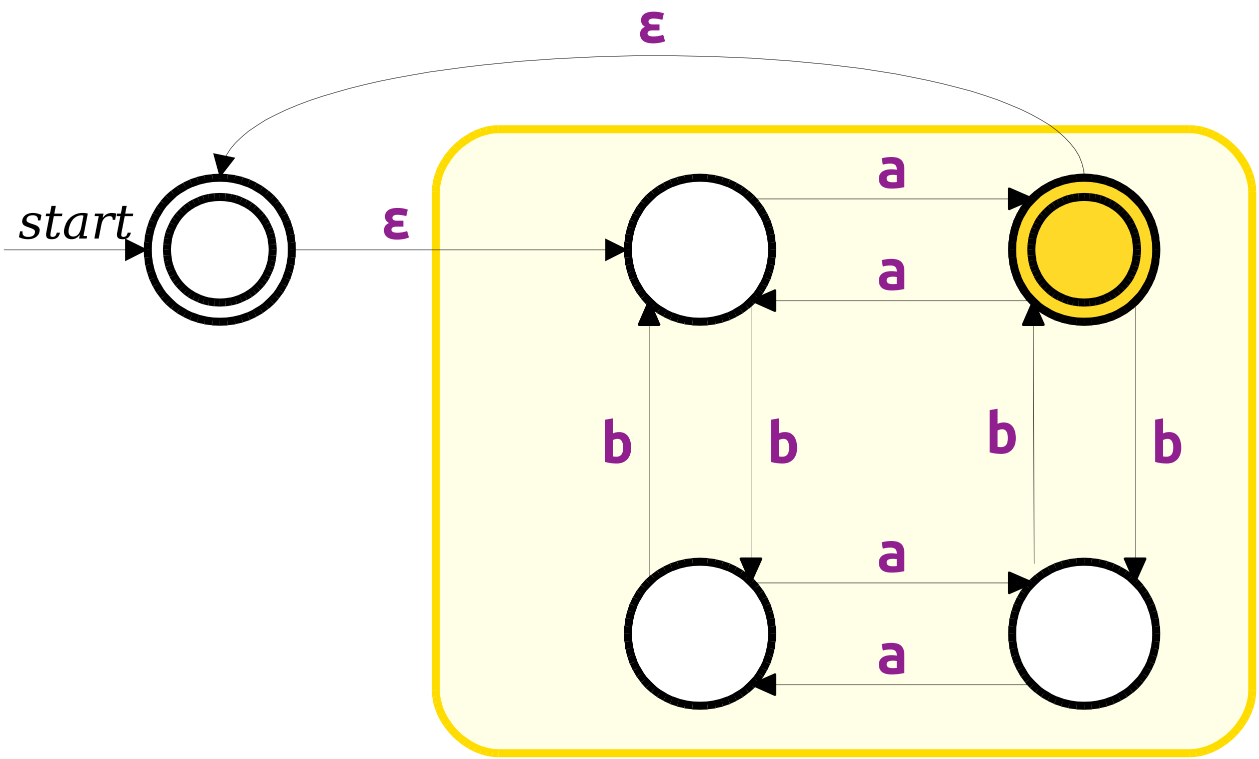

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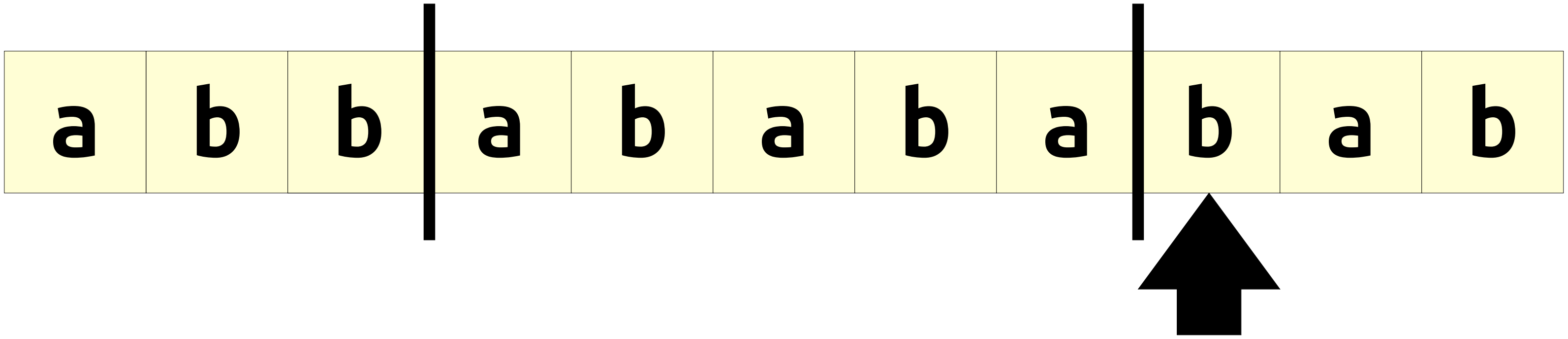

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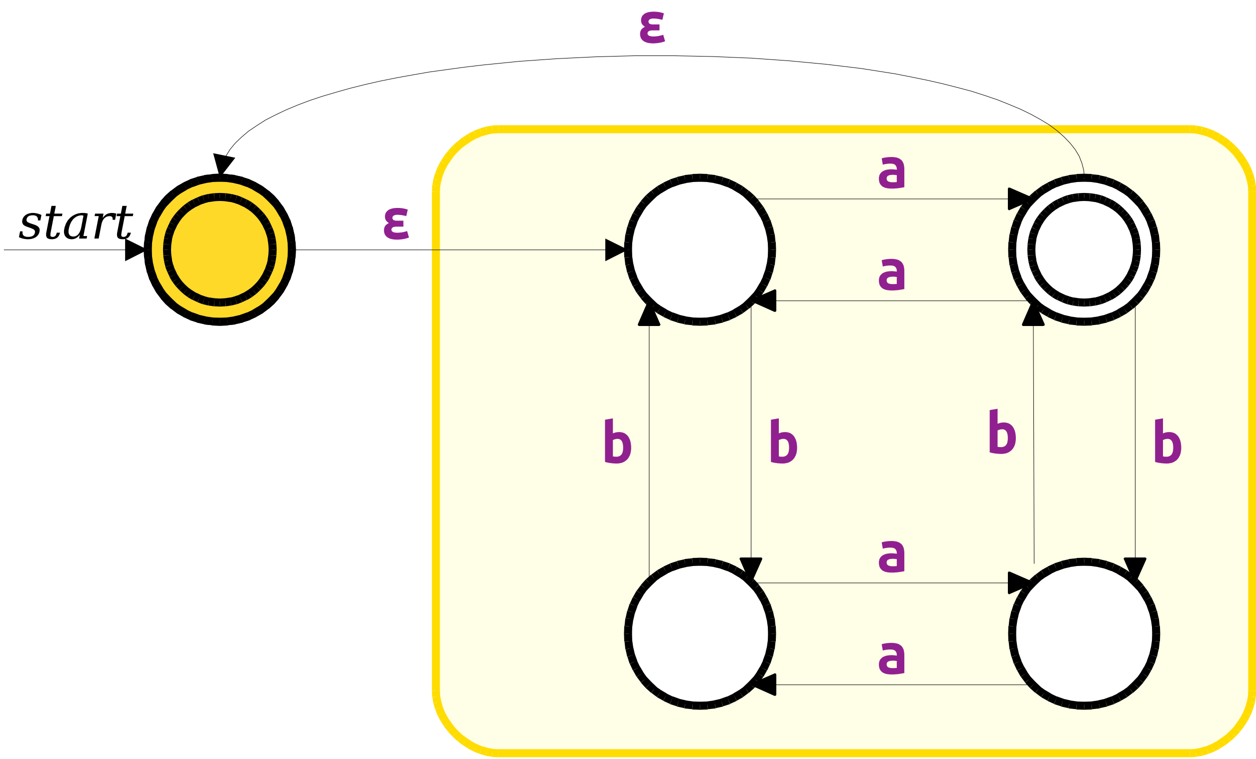

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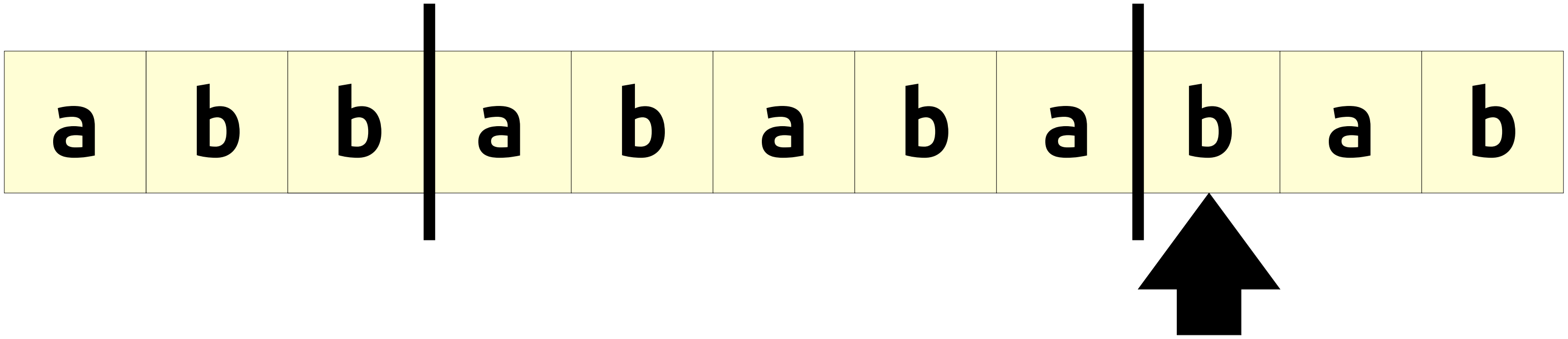

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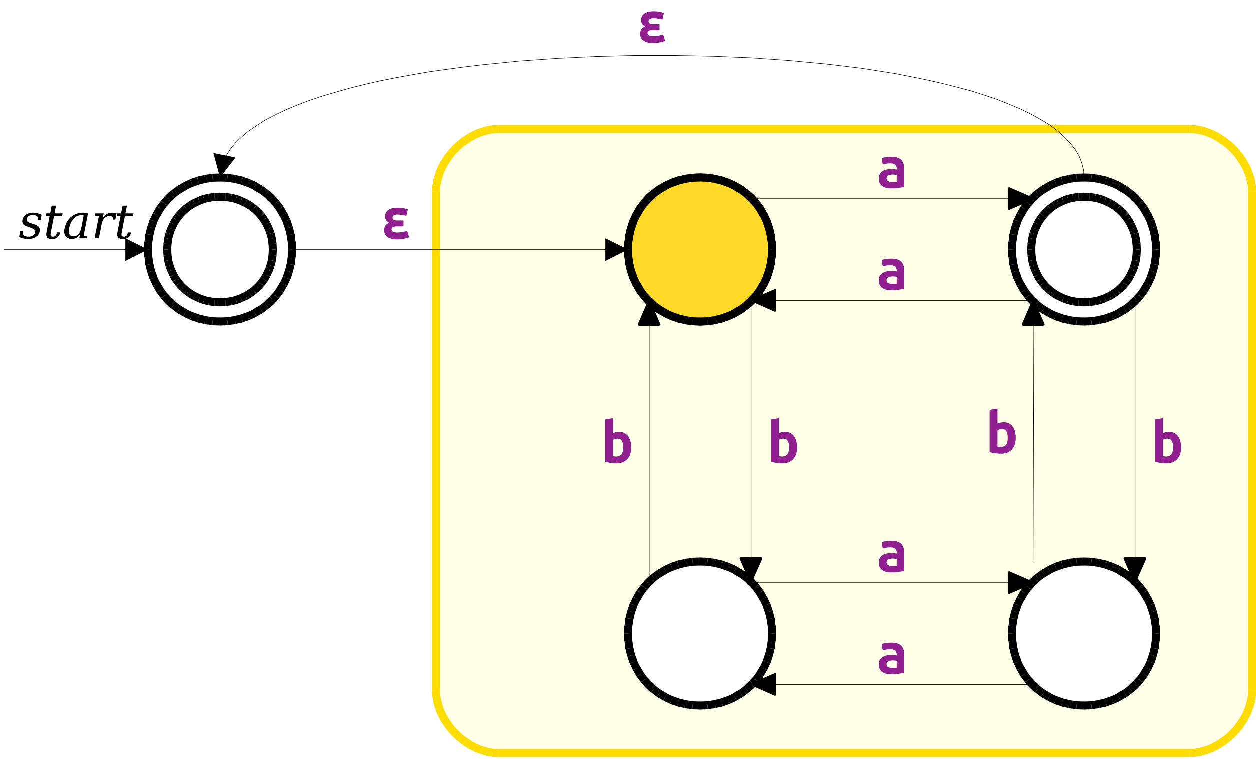

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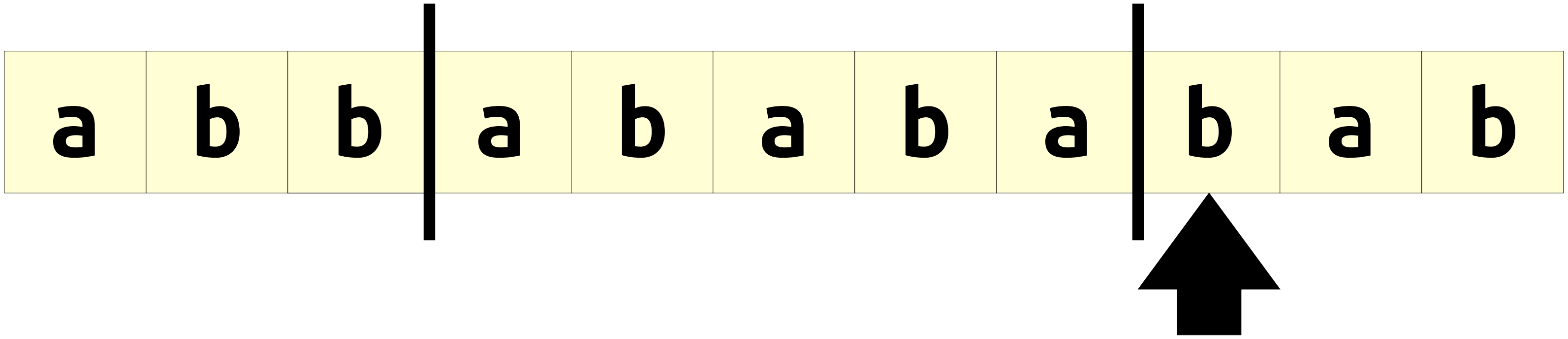

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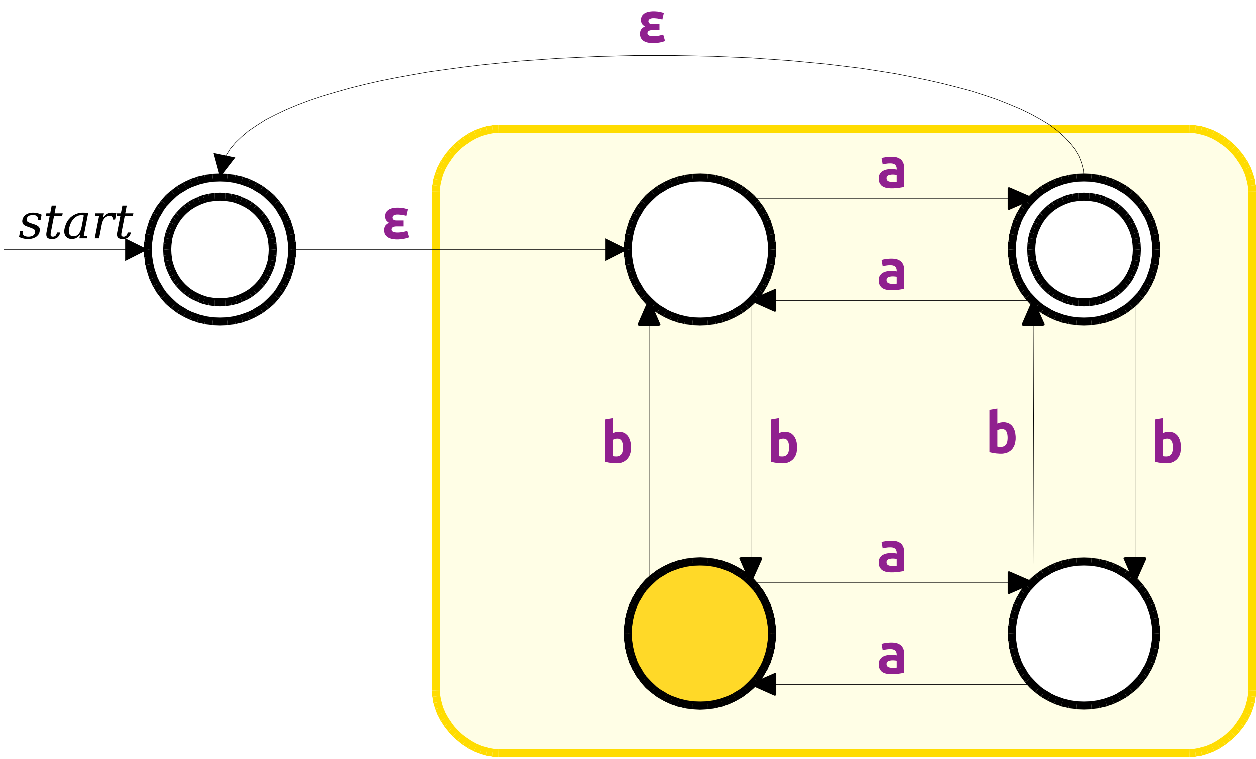

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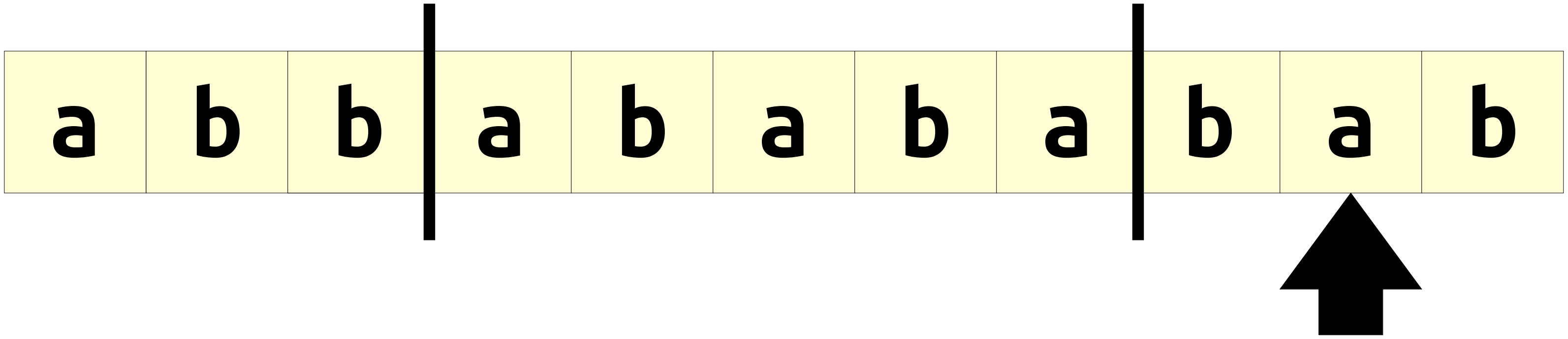

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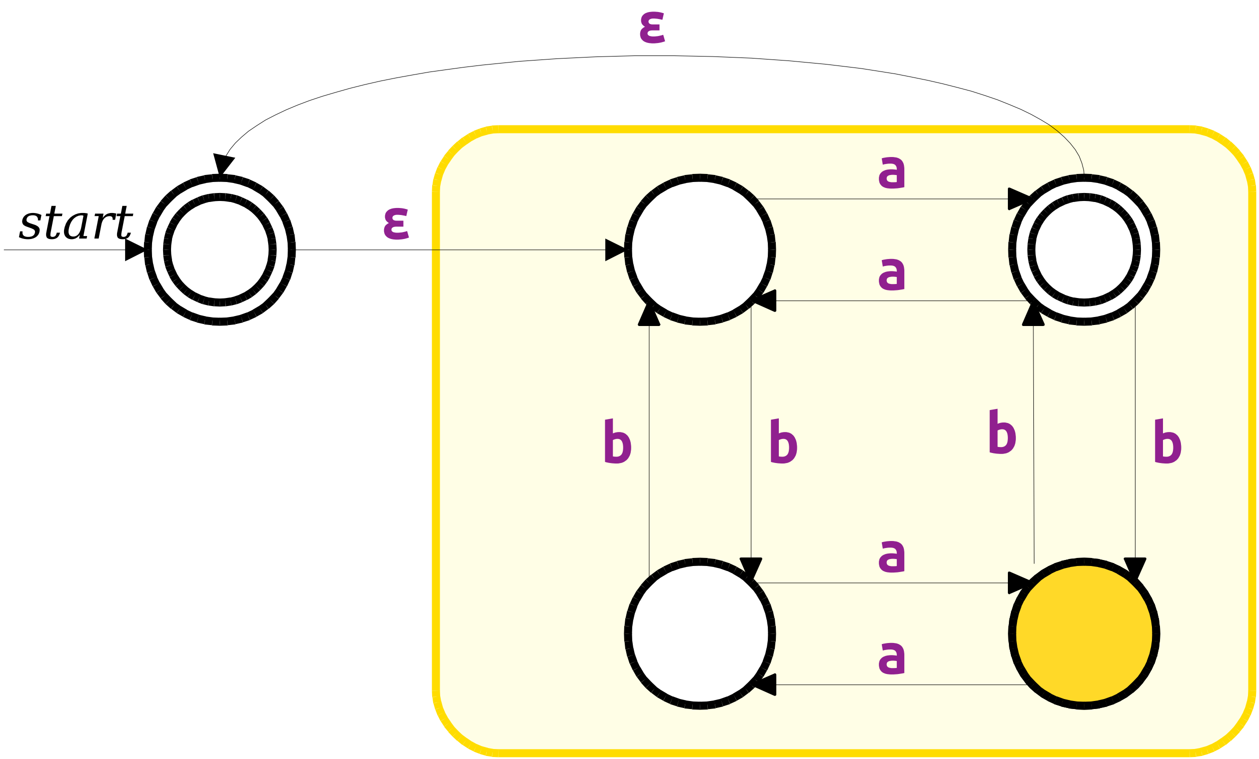

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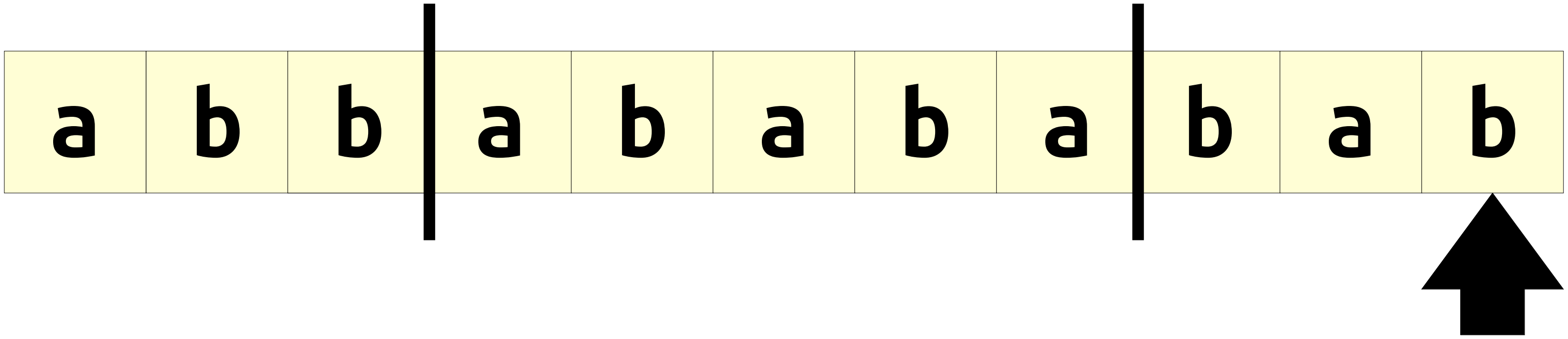

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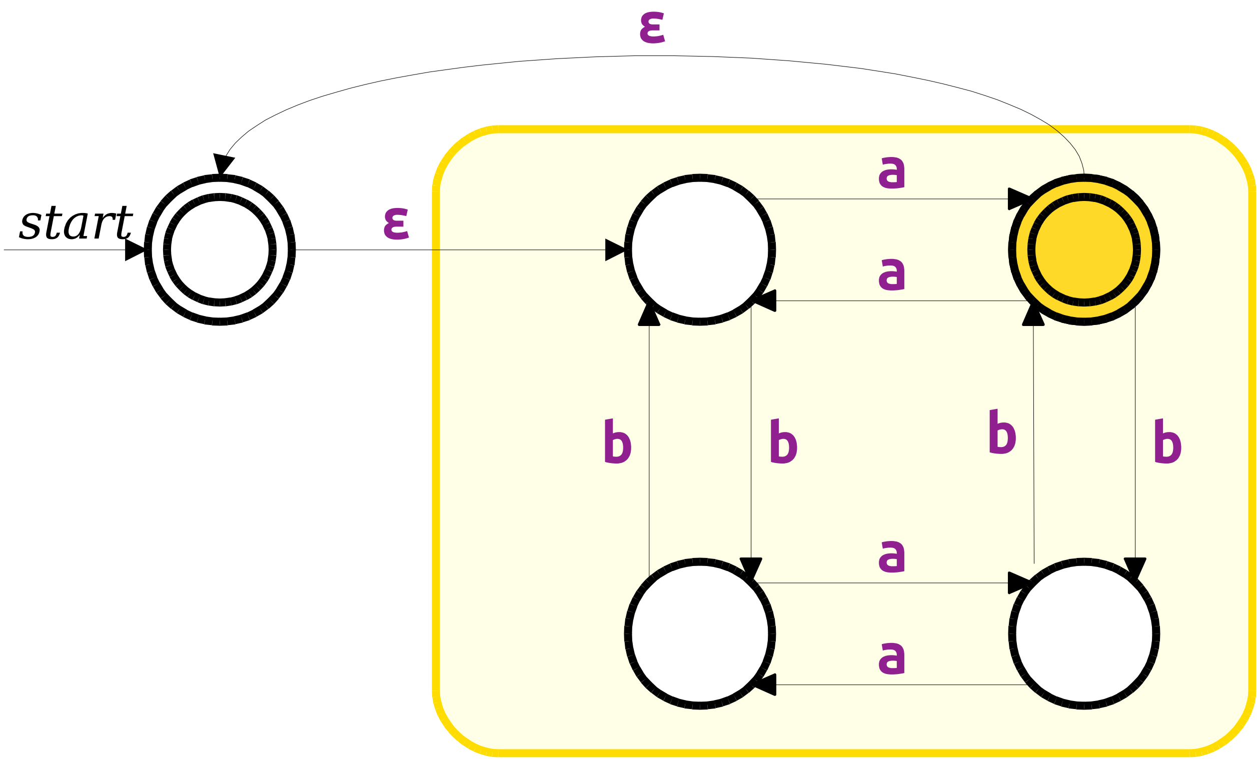

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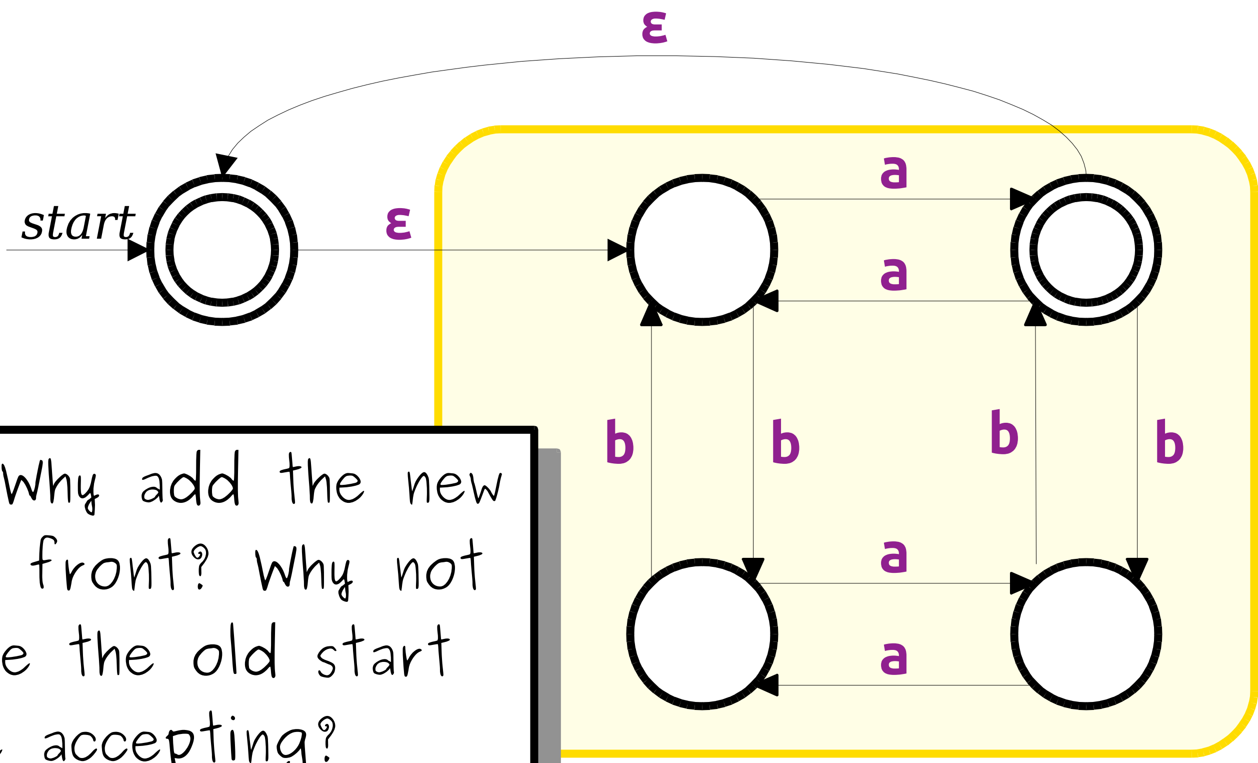

DFA for L

Question: Why add the new state out front? Why not just make the old start state accepting?

a b b | a b a b a | b a b

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Construct an NFA for L*.

Closure Properties

- ***Theorem:*** If L_1 and L_2 are regular languages over an alphabet Σ, then so are the following languages:
  - $L_1 \cup L_2$
  - $L_1 \cap L_2$
  - L_1L_2
  - L_1*
- These properties are called ***closure properties of the regular languages***.

Next Time

- ***Regular Expressions***
  - Building languages from the ground up!
- ***Thompson's Algorithm***
  - A UNIX Programmer in Theoryland.
- ***Kleene's Theorem***
  - From machines to programs!